11-13-12

1 Questions

10.5 # 3: $\sum_{n=1}^{\infty} \frac{1}{n^n}$, Guess: converge. Apply Ratio Test:

$$\begin{aligned}
&\lim_{n\to\infty} \left| \frac{1}{(n+1)^{n+1}} \frac{n^n}{1} \right| \\
&= \lim_{n\to\infty} \frac{n^n}{(n+1)^{n+1}} \\
&= \lim_{n\to\infty} \frac{n^n}{(n+1)(n+1)^n} \\
&= \lim_{n\to\infty} \frac{1}{(n+1)} \lim_{n\to\infty} \frac{n^n}{(n+1)^n} \\
&= \lim_{n\to\infty} \frac{1}{n+1} e^{-1} = 0 < 1
\end{aligned}$$

Therefore the ratio test implies the series converges absolutely. To show $\lim_{n\to\infty} \frac{n^n}{(n+1)^n}$ let $L = \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n$. Then we have

$$\begin{aligned}
\ln(L) &= \ln\left(\lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n\right) \\
&= \lim_{n\to\infty} \left(\ln\left(\frac{n}{n+1}\right)^n\right) \\
&= \lim_{n\to\infty} n \ln\left(\frac{n}{n+1}\right) \\
&= \lim_{n\to\infty} \frac{\ln(n) - \ln(n+1)}{1/n} \\
&= \lim_{n\to\infty} \frac{1/n - 1/(n+1)}{-1/n^2} \\
&= \lim_{n\to\infty} n^2(1/(n+1) - 1/n) \\
&= \lim_{n\to\infty} \left(\frac{n^2}{n+1} - n\right) \\
&= \lim_{n\to\infty} \left(\frac{n^2 - n(n+1)}{n+1}\right) \\
&= \lim_{n\to\infty} \frac{-n}{n+1} = -1
\end{aligned}$$

Therefore $\ln(L) = -1$ and hence $L = e^{-1}$.

10.5 #41: $\sum_{n=4}^{\infty} \left(1 + \frac{1}{n}\right)^{-n^2}$

Apply Root Test:to s

$$\begin{aligned}
\lim_{n\to\infty} \left[\left(1 + \frac{1}{n}\right)^{-n^2}\right]^{1/n} &= \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^{-n} \\
&= e^{-1} < 1
\end{aligned}$$

By the root test the original series converges absolutely. To see $\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^{-n} = e^{-1}$ let $L = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^{-n}$. Then proceed as in the previous problem.

2 Converge or Diverge

1. $\sum_{n=2}^{\infty} \ln\left(\frac{n}{n+2}\right)$
2. $\sum_{n=2}^{\infty} (-1)^n \frac{\ln(\ln(n))}{\ln(n)}$
3. $\sum_{n=1}^{\infty} \frac{n!(2n)!6^n}{(3n)!}$
4. $\sum_{n=2}^{\infty} \frac{\sqrt{n^5+n+1}}{\sqrt[4]{2n^{18}+n^7+4n+2}}$
5. $\sum_{n=3}^{\infty} \frac{\cos(e^{n!})}{n^2+1}$

3 Power Series

Find the radius of convergence and interval of convergence for the power series

1. $\sum_{n=1}^{\infty} \frac{x^n}{n!}$
2. $\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^2+1}$
3. $\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n3^n}$
4. $\sum_{n=0}^{\infty} \frac{n(2x-1)^{2n}}{(n^2+1)5^n}$