10-15-12

1 Questions:

2 Power Series:

Find center, radius, and interval of convergence.

1. $\sum_{n=1}^{\infty} \frac{x^n}{n!}$

$$\begin{aligned} \lim_{n\to\infty}\left|\frac{x^{n+1}}{(n+1)!}\frac{n!}{x^n}\right| &= \lim_{n\to\infty}\left|\frac{xx^n}{(n+1)n!}\frac{n!}{x^n}\right| \\ &= \lim_{n\to\infty}\left|\frac{x}{(n+1)}\right| \\ &= |x|\lim_{n\to\infty}\frac{1}{n+1} = 0 < 1 \end{aligned}$$

   For all x. Therefore the power series converges absoluetly for all values of x. The radius of convergence is $R = \infty$ and the interval of convergence $I = (-\infty, \infty)$.

2. $\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^2+1}$ Ratio Test:

$$\begin{aligned} \lim_{n\to\infty}\left|\frac{(x-2)^{n+1}}{(n+1)^2+1}\frac{n^2+1}{(x-2)^n}\right| &= \lim_{n\to\infty}\left|\frac{(x-2)(x-2)^n(n^2+1)}{[(n+1)^2+1](x-2)^n}\right| \\ &= |x-2| < 1. \end{aligned}$$

   $-1 < x-2 < 1 \Rightarrow 1 < x < 3$. Radius of convergence is $R = 1$. For interval, we need to check end points.

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+1}$$

   Is absolutely convergent (as we have seen before). $x = 3$

$$\sum_{n=0}^{\infty} \frac{1}{n^2+1}$$

   is convergent. Therefore interval of convergence is $1 \le x \le 3$. Center of the power series is $c = 2$.

3. $\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n3^n}$

4. $\sum_{n=0}^{\infty} \frac{n(2x-1)^{2n}}{(n^2+1)5^n}$