**11-27-12**

# 1 Questions:

10.7 #3: Find a Mac. Series for $f(x) = \frac{1}{1-2x}$.

$$f'(x) = \frac{2}{(1-2x)^2} \quad f'(0) = 2$$
$$f''(x) = \frac{2 \cdot 2^2}{(1-2x)^3} \quad f''(0) = 2 \cdot 2^2$$
$$f^{(3)}(x) = \frac{3!2^3}{(1-2x)^4} \quad f^{(3)}(0) = 3! \cdot 2^3$$
$$f^{(n)}(x) = \frac{n!2^n}{(1-2x)^{n+1}} \quad f^{(n)}(0) = n! \cdot 2^n$$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = \sum_{n=0}^{\infty} \frac{n! \cdot 2^n}{n!} x^n = \sum_{n=0}^{\infty} 2^n x^n = \sum_{n=0}^{\infty} (2x)^n.$$

Converges for $|2x| < 1$ or $|x| < \frac{1}{2}$.

10.7 #7: Find a Mac. series for $f(x) = \sin(x^2)$. We will first find a Mac series for $g(x) = \sin(x)$:

$$g'(x) = \cos(x) \quad \cos(0) = 1$$
$$g''(x) = -\sin(x) \quad -\sin(0) = 0$$
$$g^{(3)}(x) = -\cos(x) \quad -\cos(0) = -1$$
$$g^{(4)}(x) = \sin(x) \quad \sin(0) = 0$$
$$g^{(5)}(x) = \cos(x) \quad \cos(0) = 1$$
$$g^{(2n)}(x) = (-1)^n \sin(x) \quad g^{(2n)}(0) = 0$$
$$g^{(2n+1)}(x) = (-1)^n \cos(x) \quad g^{(2n+1)}(0) = (-1)^n$$

$$g(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$f(x) = \sin(x^2) = g(x^2) = \sum_{n=0}^{\infty} \frac{(-1)^n (x^2)^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{(2n+1)!}$$

Converges for all values of $x$ by the ratio test.

# 2 Group Problems

1. Find a Maclaurin series for $f(x) = e^{2x}$.

2. Find a Maclaurin series for $\ln(x+1)$. Use this to evaluate $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$.

3. Use an infinite series to estimate the integral $\int_0^1 e^{-x^2}\, dx$ with |error|$< 0.001$.