1 Questions:

10.7 #33: Find a Taylor series centered at 2 for $f(x) = x^4 + 3x - 1$.

$$\begin{aligned} f'(x) &= 4x^3 + 3 \;\; f'(2) = 35 \\ f''(x) &= 12x^2 \;\; f''(2) = 48 \\ f^{(3)}(x) &= 24x \;\; f^{(3)}(2) = 48 \\ f^{(4)}(x) &= 24 \;\; f^{(4)}(2) = 24 \\ f^{(n)}(x) &= 0 \;\; f^{(n)}(2) = 0 \;\; n \geq 5 \end{aligned}$$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(2)}{n!}(x-2)^n = 21 + 35(x-2) + \frac{48}{2}(x-2)^2 + \frac{48}{6}(x-2)^3 + \frac{24}{24}(x-2)^4.$$

Practice Exam #8: $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n^2+18n+7}x^n$ has radius of convergence 1. Find a power series for $f'(x)$.

$$f'(x) = \frac{d}{dx}\sum_{n=0}^{\infty} \frac{1}{n^2+18n+7}x^n = \sum_{n=0}^{\infty} \frac{d}{dx}\frac{1}{n^2+18n+7}x^n = \sum_{n=0}^{\infty} \frac{n}{n^2+18n+7}x^{n-1}$$

has radius of convergence 1 by theorem 2 from 10.6.

Practice Exam #5: Use comparision test or limit comparison test for the series: $\displaystyle\sum_{n=0}^{\infty} \frac{\sqrt{n}}{n+2}$. Limit comparision with $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n} = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

$$\begin{aligned} \lim_{n\to\infty} \frac{\sqrt{n}/(n+2)}{\sqrt{n}/n} &= \lim_{n\to\infty} \frac{n}{n+2} \\ &= 1 \end{aligned}$$

Therefore both series converge or both diverge. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverges by p-series with $p = 1/2$.

Practice Exam #6: $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{\sqrt{n}}{n+2}$. Alternating series test: $\dfrac{\sqrt{n}}{n+2} > 0$. If $f(x) = \dfrac{\sqrt{x}}{x+2}$, then $f'(x) = \dfrac{-x+2}{2\sqrt{x}(x+2)^2} < 0$ for $x > 2$. So $f(x)$ is decreasing for $x > 2$. Also we have

$$\lim_{n\to\infty} \frac{\sqrt{n}}{n+2} = 0$$

2 Group Problems:

1. Eliminate the parameter for the parametric curve defined by $x = t^{-1}$ and $y = t^{-2}$.
2. Sketch the curve $c(t) = (t^2, t^3 - 3t)$.
3. Find the equation of the tangent line for the parametric curve defined by $x = 6\sin(t)$ and $y = t^2 + t$ at the point $(0, 0)$.
4. Find the length of the curve defined by $x(t) = e^t + e^{-t}$ and $y(t) = 5 - 2t$ on the interval $0 \leq t \leq 3$.
5. Find the length of the curve defined by $x(t) = \dfrac{t}{1+t}$ and $y(t) = \ln(1+t)$ for $0 \leq t \leq 2$.