11/06/12

Series Rule:

1. Integral Test
2. Comparison Test
3. Limit comparison Test
4. Ratio Test
5. Root Test
6. Alternating Series Test
7. Test for divergence
8. Test for Absolute Convergence

Question: Does the series $\sum_{n=1}^{\infty} \frac{1}{2^n}$ converge? If it converges, can we find where?

This is a geometric series with $r = \frac{1}{2}$. Therefore the sries converges: $\frac{1/2}{1-1/2}$.

Question: section 10.5 #18. $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^3}{(3n)!}$. Determine if the series converges absolutely.

We apply the ratio test:

$$\lim_{n\to\infty} \left| \frac{((n+1)!)^3}{(3(n+1))!} \cdot \frac{(3n)!}{(n!)^3} \right| = \lim_{n\to\infty} \frac{(n+1)^3 (n!)^3 (3n)!}{(3n+3)(3n+2)(3n+1)(3n)!}$$
$$= \lim_{n\to\infty} \frac{(n+1)^3}{(3n+3)(3n+2)(3n+1)} = \frac{1}{27} < 1.$$

Note that $\frac{1}{27} < 1$. By the ratio test, the original series converges absolutely.

Question: 10.5 #42. Prove that $\displaystyle\sum_{n=1}^{\infty} \frac{2^{n^2}}{n!}$ diverges. Hint: Use $2^{n^2} = (2^n)^n$ and $n! \leq n^n$.