LINEAR CONTRASTS OR COMPARISONS

In planning an experiment, we can often provide for F tests to answer pertinent questions.

This involves partitioning the df and TRT SS.

Skillfully selected treatments can answer as many independent questions as there are TRT df.

When comparisons are independent, they are said to be **orthogonal**.

<u>Advantages of Linear Contrasts</u>

1. They enable one to answer specific, important questions about treatment effects.

2. The computations are simple.

3. They provide a useful check on the TRT SS.

<u>Orthogonal Coefficients</u>

Linear contrasts involve the selection of orthogonal coefficients.

Comparisons for coefficients are constructed using the following rules.

1. If two groups of equal size are to be compared, assign coefficients of +1 to the members of one group and -1 to those of the other group. It does not matter which group is assigned the positive coefficient.

2. In comparing groups containing different numbers of treatments, assign to the first group, coefficients equal to the number of treatments in the second group, and to the second group, coefficients of the opposite sign equal to the number of treatments in the first group.

3. Reduce coefficients to the smallest possible integers. For example, in comparing a group of two treatments with a group of four, by rule 2, the coefficients are +4,+4,-2,-2,-2. These can be reduced to +2,+2,-1,-1,-1,-1.

4. Interaction coefficients can always be found by multiplying the corresponding coefficients of the main effects.

For comparisons to be orthogonal (i.e., independent), the following must be true:

1. The sum of the coefficients must equal zero (i.e., $\Sigma c_i=0$.)
2. The sum of the product of the corresponding coefficients of any two comparisons is zero (i.e., $\Sigma c_i c_{i'}=0$).

An Example to show how to construct a table of orthogonal coefficients.

Suppose an experiment is planned to test the efficiency of phosphorus fertilization by three methods: broadcast (B), shallow band placement (S), and deep placement (D). Also, for each one of these methods of placement, two rates of phosphorus will be applied (P_1, and P_2). A nonfertilized treatment will be included (NT) to establish a response of the fertilizer.

Step 1. List the treatments across the top of the table.
Step 2. Write in the comparison coefficients as you decide what comparisons will be made. There will be t-1 comparisons.

Comparison	NT	P_1B	P_1S	P_1D	P_2B	P_2S	P_2D
Response to P	6	-1	-1	-1	-1	-1	-1
P_1 vs P_2	0	1	1	1	-1	-1	-1
B vs S+D	0	2	-1	-1	2	-1	-1
S vs D	0	0	1	-1	0	1	-1
(P_1 vs P_2)x(B vs S+D)	0	2	-1	-1	-2	1	1
(P_1 vs P_2)x(S vs D)	0	0	1	-1	0	-1	1

<u>Description of Comparisons</u>

1. Is there a response to P. (NOTE: Having made a comparison involving a single trt compared with all the rest, NT cannot be used again if the comparisons are to be orthogonal.)

2. Is the average response to P_2 greater than that of P_1?.

3. Over both levels of P, is band placement superior to broadcast?

4. Considering band placement only, is there a difference between shallow and deep?

5. Is the change in yield from P_1 to P_2 different for broadcast compared to band placement?

6. Is the change in yield from P_1 to P_2 different for shallow compared to deep band placement.

In the above table, note that coefficients of all rows sum to zero and that the sum of the coefficients for the same treatments for any two comparisons sum to zero.

For each single degree of freedom contrast, a sum of square, mean square, and an F-test can be calculated.

$$SS = (\Sigma cY_{i.})^2/(r\Sigma c_i^2) \quad = Q^2/rk$$

where:
- c = coefficients
- $Y_{i.}$ = Treatment totals
- r = number of replicates
- $Q = (\Sigma cY_{i.})^2$
- $k = \Sigma c_i^2$

Numerical Example

A study is conducted to investigate the effect of a protein supplement on male and female lamb weight gain. There are four treatments:

- P_0S_0 No protein supplement, male lamb
- P_0S_1 No protein supplement, female lamb
- P_1S_0 Protein supplement, male lamb
- P_1S_1 Protein supplement, female lamb

We can ask three pertinent questions.

1. Considering all lambs, does the protein supplement affect weight gain ability?
2. Irrespective of protein supplement, are there differences in gaining ability between male and females?
3. Does the protein supplement have the same affect on weight gain ability in male and female lambs?

Replicate	P_0S_0	P_0S_1	P_1S_0	P_1S_1	$Y_{.j}$
1	54	51	68	57	230
2	57	50	65	61	233
3	58	53	62	59	232
4	59	58	57	59	233
$Y_{i.}$	228	212	252	236	928

Step 1. Calculate correction factor

$$CF = (\Sigma Y_{ij})^2/rt = 928^2/4x4 = 53{,}824$$

Step 2. Calculate Total SS

$$\text{Total SS} = \Sigma Y_{ij}^2 - CF = 334$$

Step 3. Calculate Replicate SS

$$\text{Rep SS} = Y_{.j}^2/t - CF = 1.5$$

Step 4. Calculate Treatment SS

$$\text{Trt SS} = Y_{i.}^2/r - CF = 208.0$$

Step 5. Calculate Error SS

$$\text{Error SS} = \text{Total SS} - \text{Rep SS} - \text{Trt SS} = 124.5$$

Step 6. Partition Treatment SS into single df contrasts

	P_0S_0	P_0S_1	P_1S_0	P_1S_1
Trt Totals	228	212	252	236
Protein supplement vs. No protein supplement	+1	+1	-1	-1
Male lambs vs. Female lambs	+1	-1	+1	-1
Protein supplement x Sex of lambs	+1	-1	-1	+1

i. Write coefficients

ii. Calculate SS for each single df Comparison

$$\text{SS Protein} = (\Sigma cY_{i.})^2/(r\Sigma c_i^2) \quad = Q^2/rk$$

$$\frac{[(1)228 + (1)212 + (-1)252 + (-1)236]^2}{4[(1)^2 + (1)^2 + (-1)^2 + (-1)^2]} = 144$$

$$\text{SS Sex} = \frac{[(1)228 + (-1)212 + (1)252 + (-1)236]^2}{4[(1)^2 + (-1)^2 + (1)^2 + (-1)^2]} = 64$$

$$\text{SS Protein x sex} \frac{[(1)228 + (-1)212 + (-1)252 + (1)236]^2}{4[(1)^2 + (-1)^2 + (1)^2 + (-1)^2]} = 0$$

Step 7. Write ANOVA

Sources of variation	df	SS	MS	F
Rep	3	1.5	0.5	0.04
Trt	3	208.0	69.33	5.0*
Protein	1	144.0	144.0	10.4*
Sex	1	64.0	64.0	4.6
Protein x Sex	1	0.0	0.0	0.0
Error	9	124.5	13.83	
Total	15	334.0		

Conclusions

1)	n	Mean
No protein supplement	8	55.0
Protein supplement	8	61.0

Considering all lambs, addition of a protein supplement significantly increased weight gaining ability.

2)	n	Mean
Male	8	60.0
Female	8	56.0

Considering both diets together, there was no significant difference in weight gaining ability between males and females.

	n	Mean
Male, no supplement	4	57.0
Female, no supplement	4	53.0
Male, supplement	4	63.0
Female, supplement	4	59.0

Male and female lambs responded similarly in weight gaining ability to the addition of a protein supplement to their diet.