

## Talent Search

The Department of Mathematics at NDSU is happy to announce the start of the annual North Dakota Mathematics Talent Search. The goals of the talent search are to locate high school students in North Dakota and surrounding areas with a talent for solving mathematical problems, to reward these students and their teachers for their efforts, and to encourage these students to attend NDSU and major in the mathematical sciences or engineering.

The Talent Search poses sets of challenging mathematical problems throughout the year which will be posted on our website at

<http://math.ndsu.nodak.edu/talent/2014/>

Interested students are strongly encouraged to send in solutions even if they only solve one problem in a set; **finding a good solution to a problem is always an achievement**. The problems do not require advanced mathematical knowledge – just creativity and a feeling or taste for problem solving.

The students who submit a significant number of mathematically sound solutions for each of the three rounds will be rewarded with various prizes, including a one-year subscription to a science/mathematics magazine of their choice. The best participants who decide to attend NDSU and major in the mathematical sciences will also be rewarded with scholarships.

**Please submit your solutions by email to [maria.alfonseca@ndsu.edu](mailto:maria.alfonseca@ndsu.edu), by October 17, 2014.** Alternatively, solutions may be sent by regular mail to:

Talent Search  
c/o Maria Alfonseca  
Mathematics NDSU Dept.# 2750  
PO BOX 6050  
Fargo, ND 58108-6050

**Please do not forget to include your name, postal address, school, and e-mail address.**

Here is the first set of problems:

1. Prove that  $11^{10} - 1$  is divisible by 100.
2. A median of a triangle is a line segment from a vertex of the triangle to the midpoint of the side opposite to that vertex. Because there are three vertices, there are three possible medians. Show that it is *not* possible to have a triangle with sides  $\alpha$ ,  $\beta$  and  $\gamma$  whose medians have lengths  $\frac{2}{3}\alpha$ ,  $\frac{2}{3}\beta$  and  $\frac{4}{5}\gamma$ .
3. The sum of a certain number of consecutive positive integers is 1000. Find these numbers.

4. A complex number is a number of the form  $a + ib$ , where  $a$  and  $b$  are real numbers and  $i = \sqrt{-1}$ . Suppose  $z$  is a complex number such that

$$z^2 + z + \frac{1}{z} + \frac{1}{z^2} + 1 = 0.$$

If  $N$  is a positive integer, find the value of

$$z^{2N} + z^N + \frac{1}{z^N} + \frac{1}{z^{2N}}.$$

5. We have a pool table (without holes), measuring one meter wide and two meters long. We hit a ball placed at the center of the table, in such a way that when the ball comes back to the center for the first time, the total length it has covered is 25 meters. How many times did the ball bounce off the sides?