

Talent Search

The Department of Mathematics at NDSU is happy to announce the start of the annual North Dakota Mathematics Talent Search. The goals of the talent search are to locate high school students in North Dakota and surrounding areas with a talent for solving mathematical problems, to reward these students and their teachers for their efforts, and to encourage these students to attend NDSU and major in the mathematical sciences or engineering.

The Talent Search poses sets of challenging mathematical problems throughout the year which will be posted on our website at <http://math.ndsu.nodak.edu/talent/2014/>

Interested students are strongly encouraged to send in solutions even if they only solve one problem in a set; **finding a good solution to a problem is always an achievement**. The problems do not require advanced mathematical knowledge – just creativity and a feeling or taste for problem solving.

The students who submit a significant number of mathematically sound solutions for each of the three rounds will be rewarded with various prizes, including a one-year subscription to a science/mathematics magazine of their choice. The best participants who decide to attend NDSU and major in the mathematical sciences will also be rewarded with scholarships.

Please submit your solutions by email to maria.alfonseca@ndsu.edu, by January 16, 2015. Alternatively, solutions may be sent by regular mail to:

Talent Search
c/o Maria Alfonseca
Mathematics NDSU Dept.# 2750
PO BOX 6050
Fargo, ND 58108-6050

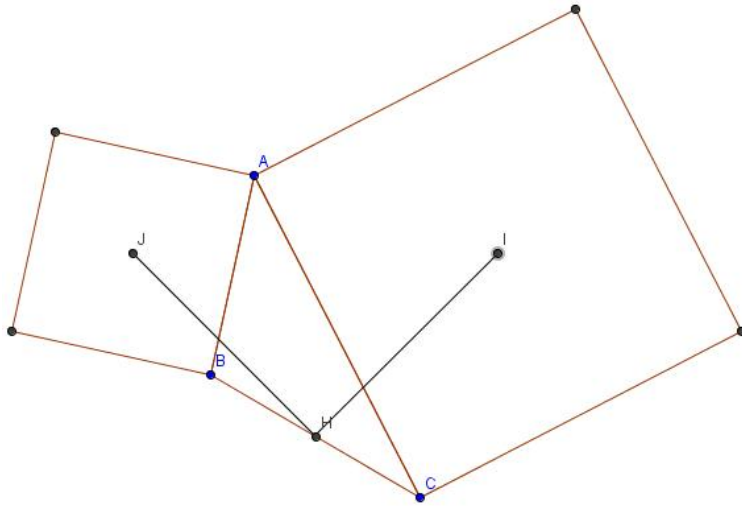
Please do not forget to include your name, postal address, school, and e-mail address.

Here is the second set of problems:

1. Find the remainder upon dividing the following number by 7:

$$10^{10} + 10^{10^2} + 10^{10^3} + \cdots + 10^{10^{10}}.$$

2. An *integral triangle* is defined as a triangle whose sides are measurable in whole numbers. Find all integral triangles whose perimeter equals their area.
3. How many zeros terminate the number which is the product of all the integers from 1 to 100, inclusive?
4. Consider a triangle with vertices ABC . We draw a square on the side AB , and another square on the side AC of the triangle, like in the figure. Let us denote the centers of the two squares by I and J . Let H be the middle point of the side BC of the triangle. Show that the segments HI and HJ have the same length and form a 90 degree angle.



5. There is a country inhabited by knights, who always tell the truth, knaves, who always lie, and normals, who sometimes tell the truth and sometimes lie. The police is investigating a crime in this country, and there are three suspects, A, B and C. The known facts are:

- Fact 1: Exactly one of them is guilty.
- Fact 2: One of them is a knight, one of them a knave, and one of them is normal. But we don't know who is what.
- Fact 3: The guilty person is not a knave.

When interrogated, the suspects said the following statements:

- A: I am innocent!
- B: A is innocent.
- C: No, A is guilty!

From these statements and the facts, the police determined that there were three possible scenarios. Find the three scenarios, determining in each of them who is the knight, who is the knave, who is normal, and who is guilty. Since they could not get any further, they called an Inspector from Scotland Yard to come help them. The Inspector asked C: "Are you guilty?" C answered something (the Inspector heard it but we did not). The Inspector thought for a moment, then asked A: "Is C guilty?" A answered, and from his answer the Inspector could determine which of the three previous scenarios was the correct one.

Who is the knight? Who is the knave? Who is normal? Who is guilty? What did C and A answer to Inspector Craig's questions?

(This problem was created by Raymond Smullyan).