Talent Search: Round 3 (Final Round)

The Talent Search poses sets of challenging mathematical problems throughout the year which will be posted on our website at
http://math.ndsu.nodak.edu/talent/2014/

Interested students are strongly encouraged to send in solutions even if they only solve one problem in a set; **finding a good solution to a problem is always an achievement**. The problems do not require advanced mathematical knowledge – just creativity and a feeling or taste for problem solving.

It is not necessary to have participated in previous rounds. Every student is encouraged to participate in this round.

The students who submit a significant number of mathematically sound solutions for each of the rounds will be rewarded with various prizes, including a one-year subscription to a science/mathematics magazine of their choice. The best participants who decide to attend NDSU and major in the mathematical sciences will also be rewarded with scholarships.

Please submit your solutions by email to maria.alfonseca@ndsu.edu, by March 27, 2015. Alternatively, solutions may be sent by regular mail to:

Talent Search
c/o Maria Alfonseca
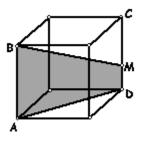Mathematics NDSU Dept.# 2750
PO BOX 6050
Fargo, ND 58108-6050

Please do not forget to include your name, postal address, school, and e-mail address.

Here is the third and last set of problems:

1. Amy is reading a book that has between 300 and 600 pages. If she reads 6 pages per day, she will have 3 pages left to read on the last day. If she reads 7 pages per day, on the last day there will be 5 pages left. How many pages does the book have? Give all the possibilities.

2. Find the smallest positive integer n such that one-half n is a perfect square, one-third n is a perfect cube and one-fifth n is a perfect fifth power.

3. Let a and b be positive integers. Show that the square root of 2 always lies between the two fractions a/b and $(a + 2b)/(a + b)$.

4. In the picture of the cube below, CM = 2MD. The area of the shaded region is 60.34 cm2. What is the volume of the cube?

5. To a person standing on the surface of Saturn, from what latitude will Saturn's rings appear to be widest? See attached picture: We assume that Saturn is perfectly spherical with a radius of 60,000 km, and that the rings are perfectly circular. The distance from the center of Saturn to the inner ring is 67,000 km, and the distance from the center of Saturn to the outer ring is 140,000 km. The latitude is the angle φ above the equator, and the rings will appear widest when the angle α is biggest.