Instructions. Answer **any 4 short questions, and any 4 long questions**. Clearly mark which questions you wish to be graded on this sheet, or else 1-4 and 6-9 will be graded. Show all work, and explain your answers clearly. Answers will be graded on correctness and clarity. All solutions should include some explanation.

Shorter questions: (5 points each)

1. Let $[n]$ denote the chain poset with n elements. Find a formula for the number of linear extensions of the disjoint union poset $[n] + [m]$.
2. Is the poset $a \lessdot b \lessdot e, a \lessdot c \lessdot e$, $a \lessdot d \lessdot e$ a distributive lattice? Why or why not?
3. What pair of tableaux (P, Q) corresponds to the following matrix by the Robinson-Schensted-Knuth correspondence? $\begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 0 \\ 2 & 1 & 0 \end{pmatrix}$
4. Use the Littlewood-Richardson rule to compute the coefficient $c^{\nu}_{\lambda,\mu}$ for $\lambda = 31$, $\mu = 431$, and $\nu = 4332$.
5. Compute the irreducible representation character χ^{λ}_{μ} where $\lambda = 4322$ and $\mu = 4331$.

Longer questions: (10 points each)

6. State the fundamental theorem of finite distributive lattices. Give an example of the correspondence in this theorem and an idea of some of the key parts of the proof.
7. Calculate the number of standard Young tableaux of shape (n, n) in two ways: by the hook-length formula and by a bijection to another set of combinatorial objects whose formula you know.
8. For which positive integers are there no connected poset P with exactly n chains? Justify your answer.
9. Let p_λ denote the power sum symmetric function indexed by a partition λ. Use the fact that $\langle p_\lambda, p_\mu \rangle = z_\lambda \delta_{\lambda,\mu}$ (where $z_\lambda = \prod_r r^{m_r}(m_r)!$, m_r is the number of times r occurs in λ, and $\delta_{\lambda,\mu} = 1$ if $\lambda = \mu$ and 0 otherwise) to show the symmetric function inner product $\langle\ ,\ \rangle$ is positive definite. Recall that positive definite means the following hold for any symmetric polynomial f:
   - $\langle f, f \rangle \geq 0$, and
   - $\langle f, f \rangle = 0$ if and only if $f = 0$.
10. Find three rows of the table of characters for the irreducible representations of S_4. Give reasons for each character value; writing the table from memory without justifying your computations will receive no credit.