MATH 857: Fourier Analysis (Spring 2017)
MWF 10:00-10:50 Morrill 101

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Office Hours: MWF 12:00-12:50 am and by appointment.
Class website: https://www.ndsu.edu/pubweb/∼alfonsec/fourier-conv.html

Topics: This course combines a short introduction to Fourier Analysis with the classic geometry of convex bodies. We will use tools such as the Fourier transform, spherical harmonics and the Radon transform to obtain geometric results about convex bodies in $\mathbb{R}^n$. The focus of the geometric study is tomography, or the reconstruction of a convex body in $\mathbb{R}^n$ from lower dimentional information about it, such as its shadows (projections), or its sections passing through a fixed point of the body. There are many interesting results (and many still-open questions), such as:

- If we know the area (but not the shape) of all the shadows of the body, can we uniquely reconstruct the body?
- If every hyperplane through the origin divides K into two parts of equal volume, must K be centrally symmetric?
- If each section of a body K by a hyperplane passing through the origin has smaller area than the the corresponding section of a body L, does K have smaller volume than L?
- If in each direction, each shadow of K fits inside the shadow of L after a rigid motion, does it mean that K itself can fit in L after a rigid motion?

Along the way, we will study certain classes of convex and star bodies, including bodies of constant width, bodies of constant brightness, zonoids and intersection bodies.

References: I will take material from several sources (all books are available in the library).

1. **Spherical Harmonics Applied to Geometry**. Please google "schneider use of spherical harmonics", click on the first result and print the pdf.
2. **Fourier Series and Fourier Transform**
   - *Harmonic Analysis: From Fourier to Wavelets*, by M.C. Pereyra and L. A. Ward, Student Mathematical Library Vol 63, American Mathematical Society.
   - *Fourier Analysis: an introduction*, by E.M. Stein and R. Shakarchi, Princeton University Press.
   - *Fourier Analysis*, by J. Duoandikoetxea, Graduate Studies in Mathematics Vol 29, American Mathematical Society.
3. **Fourier Analysis applied to Geometry**
   - *Fourier Analysis on Convex Geometry*, by A. Koldobsky, Mathematical Surveys and Monographs Vol 116, American Mathematical Society.
   - *The Interface between Convex Geometry and Harmonic Analysis*, by A. Koldobsky and V. Yaskin, CBMS Vol 108, American Mathematical Society.

Attendance: Attendance is expected and required. There will be weekly student presentations. You are responsible for all the material covered in class and all the assignments and announcements made. If you need to miss class due to sickness or other reason, please email me.

Grading policy: There will be no final exam. The grade will be based on weekly class presentations and on three sets of problems that will be collected on the following dates.

- Wednesday, February 22.
- Wednesday, March 28.
- Wednesday, April 25.

The overall grade will be calculated according to the following rule:

- Class presentations: 35%
- Each Problem Set: 20%
- Colloquium Attendance: 5%

Special Needs: Any students with disabilities or other special needs, who need special accommodations in this course are invited to share these concerns or requests with the instructor and contact the Disability Services Office as soon as possible.

Veterans and student soldiers with special circumstances or who are activated are encouraged to notify the instructor in advance.

Academic Honesty: The academic community is operated on the basis of honesty, integrity, and fair play. NDSU Policy 335: Code of Academic Responsibility and Conduct applies to cases in which cheating, plagiarism, or other academic misconduct have occurred in an instructional context. Students found guilty of academic misconduct are subject to penalties, up to and possibly including suspension and/or expulsion. Student academic misconduct records are maintained by the Office of Registration and Records. Informational resources about academic honesty for students and instructional staff members can be found at www.ndsu.edu/academichonesty.