MATH 857:
Uniqueness Problems in Harmonic and Geometric Analysis
(Spring 2023)
MWF 9:00-9:50 Minard 308

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Office Hours: MWF 10:00-10:50 am and by appointment.

Topics: This course we will study several uniqueness questions in Analysis and Geometry, using Fourier analysis and spherical harmonics. The prototype uniqueness question in this course is the isoperimetric problem, which we will first address on the plane using Fourier analysis. We will next use the plane isoperimetric inequality to solve geometric questions about convex bodies in three dimensions.

Techniques of spherical harmonics have been successfully applied by R. Schneider to solve uniqueness problems about convex bodies. We will do a short introduction to spherical harmonics and the Funk-Hecke theorem, and then apply them to solve questions on bodies whose sections have constant volume.

Some very useful techniques in geometry are the symmetrization processes, which preserve certain properties of the convex bodies while increasing their symmetries and allowing to create sequences converging to the Euclidean ball. The isoperimetric problem is once again the prototype, but there are other questions that can be solved by symmetrization, such as the two and three dimentional Mahler problem.

The references for this course are:

- Groemer, *Geometric Applications of Fourier Series and Spherical Harmonics* (Encyclopedia of Mathematics and its Applications, 61, Cambridge University Press, 1996).

- Schneider notes available here:
  http://home.mathematik.uni-freiburg.de/rschnei/Vortr%E4ge.Kent.pdf

Grading policy: There will be no final exam. The grade will be based on homework, class presentations and colloquium attendance.
The overall grade will be calculated according to the following rule:

- Class presentations: 50%

- Homework: 40%

- Colloquium Attendance: 10%

Attendance: Attendance is expected. You are responsible for all the material covered in class and all the assignments and announcements made. However, you should not come to class if you are ill. If you need to miss class due to sickness or other reason, please email me.

Special Needs: Any students with disabilities or other special needs, who need special accommodations in this course are invited to share these concerns or requests with the instructor and contact the Disability Services Office as soon as possible.

Veterans and student soldiers with special circumstances or who are activated are encouraged to notify the instructor in advance.

Academic Honesty: The academic community is operated on the basis of honesty, integrity, and fair play. NDSU Policy 335: Code of Academic Responsibility and Conduct applies to cases in which cheating, plagiarism, or other academic misconduct have occurred in an instructional context. Students found guilty of academic misconduct are subject to penalties, up to and possibly including suspension and/or expulsion. Student academic misconduct records are maintained by the Office of Registration and Records. Informational resources about academic honesty for students and instructional staff members can be found at www.ndsu.edu/academichonesty.