Reference Sheet for Applied Electromagnetics - ECE 351

Coordinate Systems:

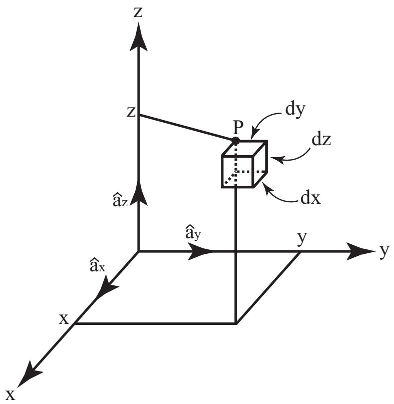

Fig. 1. The cartesian coordinate system.

The Cartesian coordinate system (x, y, z)*:*

$$\bar{A} = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z \tag{1}$$

$$d\bar{L} = dx\hat{a}_x + dy\hat{a}_y + dz\hat{a}_z \tag{2}$$

$$d\bar{S} = dydz(\pm\hat{a}_x) \tag{3}$$

$$d\bar{S} = dxdz(\pm\hat{a}_y) \tag{4}$$

$$d\bar{S} = dxdy(\pm\hat{a}_z) \tag{5}$$

$$dV = dxdydz \tag{6}$$

$$\nabla V = \frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z \tag{7}$$

$$\nabla \cdot \bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \tag{8}$$

$$\nabla \times \bar{A} = \left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right]\hat{a}_x \tag{9}$$

$$+ \left[\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right]\hat{a}_y \tag{10}$$

$$+ \left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]\hat{a}_z$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \tag{11}$$

The Cylindrical coordinate system (ρ, ϕ, z)*:*

$$\bar{A} = A_\rho\hat{a}_\rho + A_\phi\hat{a}_\phi + A_z\hat{a}_z \tag{12}$$

$$d\bar{L} = d\rho\hat{a}_\rho + \rho d\phi\hat{a}_\phi + dz\hat{a}_z \tag{13}$$

$$d\bar{S} = \rho d\phi dz(\pm\hat{a}_\rho) \tag{14}$$

$$d\bar{S} = d\rho dz(\pm\hat{a}_\phi) \tag{15}$$

$$d\bar{S} = \rho d\rho d\phi(\pm\hat{a}_z) \tag{16}$$

$$dV = \rho d\rho d\phi dz \tag{17}$$

$$\nabla V = \frac{\partial V}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\hat{a}_\phi + \frac{\partial V}{\partial z}\hat{a}_z \tag{18}$$

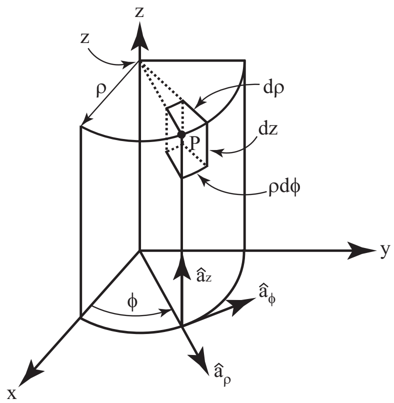

Fig. 2. The cylindrical coordinate system.

$$\nabla \cdot \bar{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z} \tag{19}$$

$$\nabla \times \bar{A} = \big[\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\big]\hat{a}_\rho \tag{20}$$

$$+ \big[\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\big]\hat{a}_\phi \tag{21}$$

$$+ \frac{1}{\rho}\big[\frac{\partial}{\partial \rho}(\rho A_\phi) - \frac{\partial A_\rho}{\partial \phi}\big]\hat{a}_z$$

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\big(\rho\frac{\partial V}{\partial \rho}\big) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} \tag{22}$$

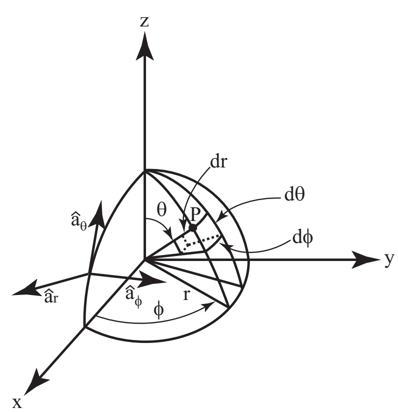

Fig. 3. The spherical coordinate system.

The Spherical coordinate system (r, θ, ϕ)*:*

$$\bar{A} = A_r\hat{a}_r + A_\theta\hat{a}_\theta + A_\phi\hat{a}_\phi \tag{23}$$

$$d\bar{L} = dr\hat{a}_r + rd\theta\hat{a}_\theta + r\sin\theta d\phi\hat{a}_\phi \tag{24}$$

$$d\bar{S} = r^2\sin\theta d\theta d\phi(\pm\hat{a}_r) \tag{25}$$

$$d\bar{S} = r\sin\theta drd\phi(\pm\hat{a}_\theta) \tag{26}$$

$$d\bar{S} = rdrd\theta(\pm\hat{a}_\phi) \tag{27}$$

$$dV = r^2\sin\theta drd\theta d\phi \tag{28}$$

$$\nabla V = \frac{\partial V}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{a}_\phi \tag{29}$$

$$\nabla \cdot \bar{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) \quad (30)$$
$$+ \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta \sin\theta) \quad (31)$$
$$+ \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$\nabla \times \bar{A} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial\phi}\right]\hat{a}_r \quad (32)$$
$$+ \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial}{\partial r}(rA_\phi)\right]\hat{a}_\theta \quad (33)$$
$$+ \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right]\hat{a}_\phi$$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) \quad (34)$$
$$+ \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) \quad (35)$$
$$+ \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2}$$

Coordinate System Transformations:

Cylindrical ⇒ Cartesian Conversion

$$x = \rho\cos\phi \quad (36)$$
$$y = \rho\sin\phi \quad (37)$$
$$z = z \quad (38)$$
$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

Cartesian ⇒ Cylindrical Conversion

$$\rho = \sqrt{x^2+y^2} \quad (39)$$
$$\phi = \tan^{-1}\left(\frac{y}{x}\right) \quad (40)$$
$$z = z \quad (41)$$
$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

Spherical ⇒ Cartesian Conversion

$$x = r\sin\theta\cos\phi \quad (42)$$
$$y = r\sin\theta\sin\phi \quad (43)$$
$$z = r\cos\theta \quad (44)$$
$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

Cartesian ⇒ Spherical Conversion

$$r = \sqrt{x^2+y^2+z^2} \quad (45)$$
$$\theta = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right) = \cos^{-1}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right) \quad (46)$$
$$\phi = \tan^{-1}\left(\frac{y}{x}\right) \quad (47)$$
$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

Transmission Lines:

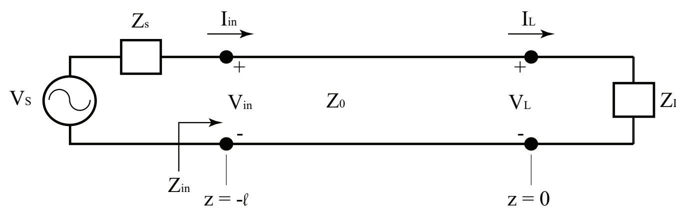

Fig. 4. Transmission line.

In General:
$$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z} \quad (48)$$
$$I_s(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z} = \frac{V_s(z)}{Z_0} \quad (49)$$
where
$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = \alpha + j\beta. \quad (50)$$
$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = |Z_0|e^{j\theta}. \quad (51)$$
Reflection at the load:
$$\Gamma = \frac{V_{0r}}{V_{0i}} = \frac{Z_L - Z_0}{Z_L + Z_0} = |\Gamma|e^{j\phi_\Gamma}. \quad (52)$$
The position of the voltage maximum is denoted as z_{max} and the position of the voltage minimum is denoted as z_{min}: $z_{min} = -\frac{1}{2\beta}[\phi_\Gamma + (2m+1)\pi]$ and $z_{max} = -\frac{1}{2\beta}[\phi_\Gamma + 2m\pi]$ where $m = 0, 1, 2....$
$$s = \frac{V_{sT}(z_{max})}{V_{sT}(z_{min})} = \frac{1+|\Gamma|}{1-|\Gamma|}. \quad (53)$$
For a finite length TL:
$$Z(z) = \frac{V_{sT}(z)}{I_{sT}(z)} \quad (54)$$
$$= Z_0\left[\frac{Z_L\cos\beta z - jZ_0\sin\beta z}{Z_0\cos\beta z - jZ_L\sin\beta z}\right].$$

Permittivity of free space: $\varepsilon_0 = 8.854187817 \times 10^{-12}$ F/m
Permeability of free space: $\mu_0 = 4\pi \times 10^{-7}$ H/m.