
# Lecture 5

## Fundamentals of Physics

## Phys 120, Fall 2014

## The secret of planetary motion: Gravity

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# Overview

- Do Newton's laws apply to planetary motion?
- Newton's theory of gravity
- Gravitational collapse: birth of the solar system
- Gravitational collapse: death of more massive stars
- Newtonian worldview: a democratic, mechanical universe
- Beyond Newton: Limitations of Newtonian Physics

## Aristotle to Newton's laws – review

Aristotle had described three laws of motion: natural motion, violent motion and celestial motion. We saw in the last lecture that Newton's approach unified natural motion and violent motion.

But what about celestial motion? Kepler carefully observed the motion of the planets and showed that the observed motion of the planets on the firmament could be derived from planets moving along elliptical orbits around the sun.

This leaves the question: what kind of force could cause this kind of motion?

## Concept check

Do you think that it would be reasonable to describe celestial motion using Newton's laws from last lecture?

What ingredient are we missing that we would need to do that?

How would you go about describing celestial motion?

# Kepler's failed attempt

Kepler had found that planets move on ellipses where the sun sits in one of the foci of the ellipse. But what keeps the planets in their orbits?

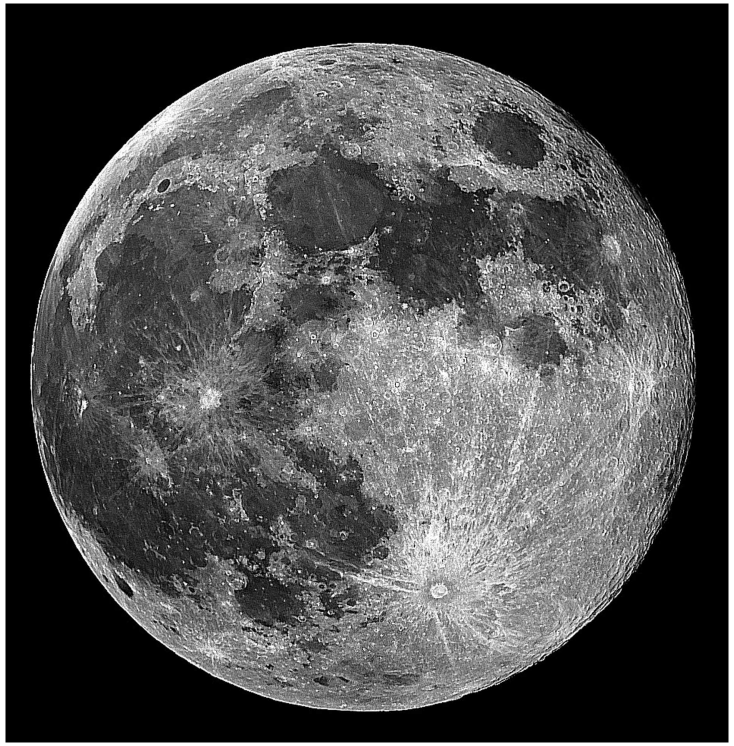
The ancient Greek notion of particles moving on crystal spheres does no longer fit.

Gilbert (1544-1603) had experimented with magnets and observed that they can act at a distance! Kepler was fascinated by this idea and thought that there must be some kind of "magnetic" force between the sun and the planets that keeps them in their path. But he did not understand the law of inertia, and thought that this force must keep pushing the planets. He was never able to consolidate this idea into a working hypothesis.

# Descartes 1596 - 1650


Rene Descartes (a contemporary of Galileo) was an important French philosopher, who developed the idea that the world could be understood in terms of mathematics. While this may be difficult in practice, the vision that this is *possible* was a huge inspiration for generations to come.

# Newton's gravity (1642 - 1727)

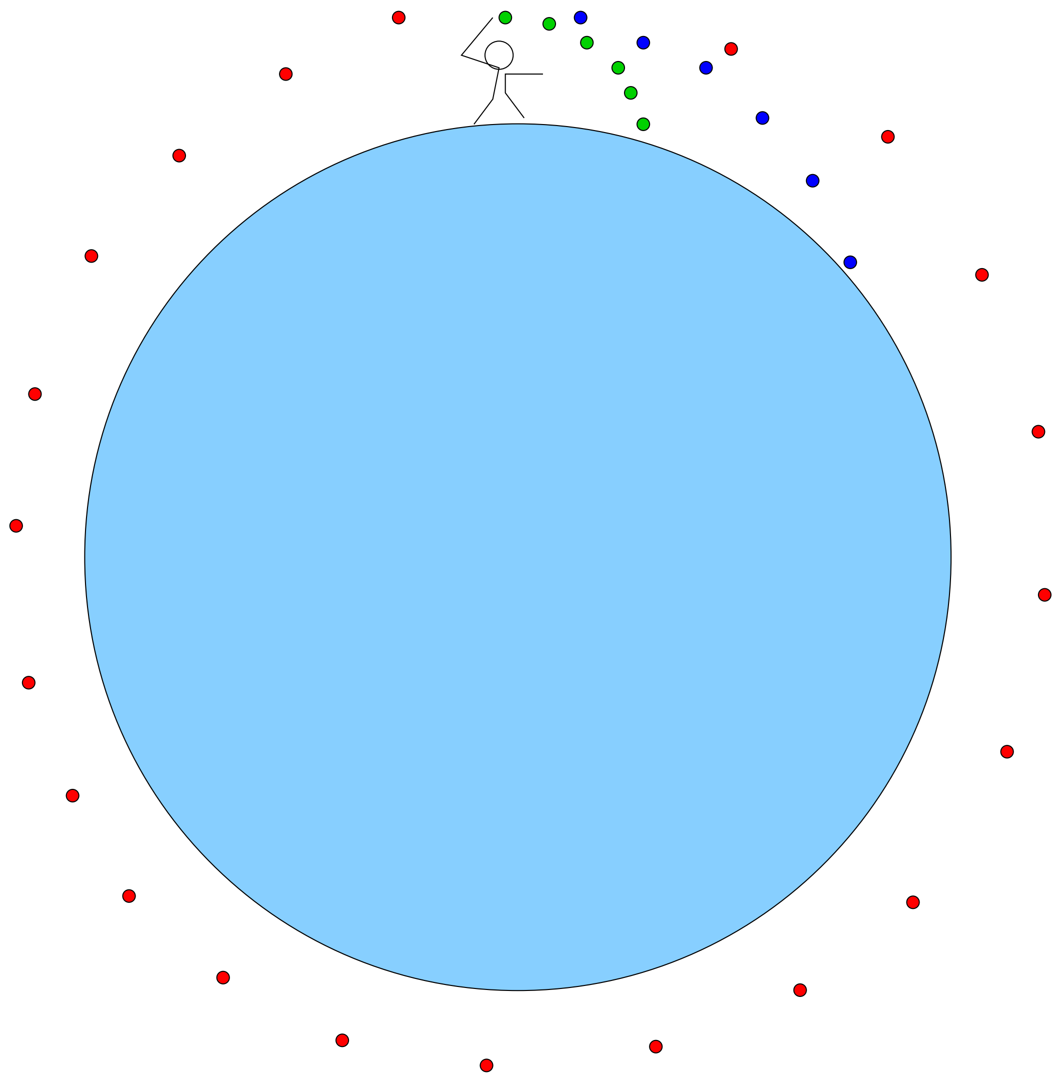


Newton reminisced at the end of his life about an experience of seeing an apple fall. There must be a force accelerating the apple, and that force is its own weight.

But what about the moon? Is it falling in the same way?

Imagine throwing an apple. The faster you throw it the further it will go, until you end up missing the earth, because you throw the apple past the horizon (neglect air resistance). That is what the moon does.

# Throwing a ball and missing the earth



Thought experiment: if you throw a ball harder and harder it will fly farther and farther. Eventually it should miss the earth and make it round to its point of origin (like a satellite).

# What is the force of gravity?

It should depend on the masses. From the apple we know that if I double the mass of the one object the force should also double. And it should be "democratic", i.e. it should depend on both masses ($m_1$ and $m_2$) in the same manner:

$$F \propto m_1 m_2$$

It also has to depend on the distance. We don't feel the gravity of single stars outside our solar system, so the force has to decay with distance. But by how much?

# Force of gravity: distance

Newton knew that moon's distance from the earth is about 60 times the earth radius. He also know how fast the moon was going (rotating around the earth every 28 days). From that he could calculate the acceleration of 0.0027 $m/s^2$ which is equal to

$$\frac{1}{60^2}g$$

where $g$ is the acceleration due to gravity on the surface of the earth. This suggested that

$$F = G\frac{m_1 m_2}{r^2}$$

where finding $G$ was still a challenge. Later it was found that $G = 6.7 \times 10^{-11} m^3/(kg\ s^2)$.

This *Gravitational constant is still the constant that is still known to the least accuracy compared to other natural constants observed in nature.*

## Conceptual questions

What happens to your weight if you double the mass of the earth?

What happens to your weight if you compress the earth to half its radius?

What happens to your weight on Mount Everest (10km above sea level)? ($R_e \approx 6400$ km)

What happens to your weight 6400 km above the surface?

# Is Newton's theory of gravity correct?

What convinced Newton (and more importantly his contemporaries) that his new theory of gravity is correct? He claimed that the force of gravity is

$$F_G = \frac{m_1 m_2}{r_{1,2}^2}$$

pointing in the direction of the other mass.

Answer: When we combine this law of gravity with his law of motion:

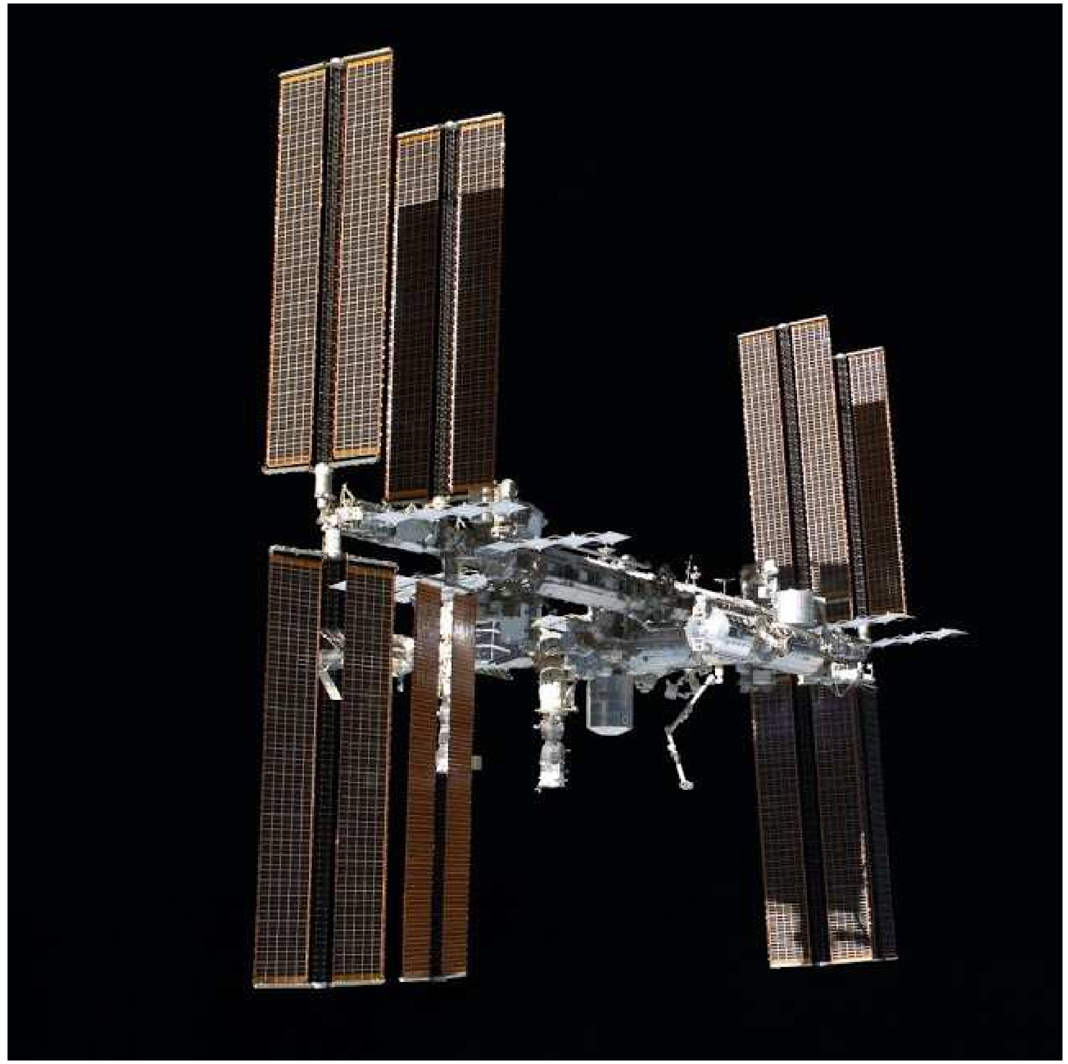
$$F = ma \tag{2}$$

we can solve this to give a prediction of the motion of the plantes. The result is that two masses, under the attraction of gravity, will rotate around a common center of gravity, on **elliptical** orbits. These are exactly the orbits that Kepler observed!

Once this fact became known, it became clear that Newton had reached an important milestone in our understanding of the world. The fact that humanity could predict the motion of the planets, more than anything, lead to a shift in the consciousness in the western world:

*A magical world is replaced with a mechanical clockwork.*

# Weightlessness in space



Why do astronauts appear weightless in the ISS, which is about 370 km above the earth?

(YouTube movie)

# Answer

They are falling, just like the apple was falling.

You know that feeling from riding in an elevator.

## How do our results fit together? (class discussion)

We have talked about the effect of gravity on our own planet. Here it has the effect of accelerating a free falling object uniformly (to a very good approximation).

$$F_g = mg$$

where $m$ is the mass of the object and $g \approx 10m/s^2$ is the acceleration due to gravity.

We also talked about massive objects much further apart. Then the law of gravity is

$$F_g = G\frac{m_1 m_2}{r^2}$$

This are two very different formulas, how do they fit together?

# Answer:

If you drop and object from a height of 1 km onto the surface of the earth, then the Force will change according to Newtons law of gravity. However, this change is so small that we cannot detect it easily.

The force of an object with weight $m = 1kg$ at height 0 (from the surface will be given by)

$$F_g = G\frac{mM}{R_e^2} = 6.7 \cdot 10^{-11}\frac{m^3}{kg\ s^2}\frac{1kg \cdot 5.97219 \cdot 10^{24}kg}{(6.371 \cdot 10^6 m)^2} = 9.8580998\frac{kg\ m}{s^2}$$

The force on the same object at height $1000m$ from the surface will be given by

$$F_g = G\frac{mM}{(R_e + 1000m)^2} = 6.7 \cdot 10^{-11}\frac{m^3}{kg\ s^2}\frac{1kg \cdot 5.97219 \cdot 10^{24}kg}{(6.372 \cdot 10^6 m)^2} = 9.8580058\frac{kg\ m}{s^2}$$

This is why we can work with $F = mg$ with the approximate value of $g \approx 10m/s^2$ for problems close to the surface of the earth.

# Other effects of gravity

The effect of weight on a planet and gravitational attraction between planets and their sun were the focus of our lecture so far, there are many important problems involving gravity that we have not yet touched on.

Can you think of any?
Where should one hope to be able to find situations where gravity is important?
Would gravity be more important for small or for big systems? Why?

# Gravitational collapse: birth of stars and planets

Clouds of hydrogen gas are slowly collapsing due to the gravitational attraction of matter. Initiated by a pressure wave matter nucleates leading to the formation of stars and planets.

# A Star is born

Gas nucleates and the protostar attracts more matter that is absorbed by the star. Eventually the star is dense and hot enough to start a nuclear fusion reaction of helium (one proton) that (eventually) leads to the formation of helium (two protons and two neutrons). A large amount of energy is released, which is the source of energy in a star. The concepts of protons, fusion, formation of elements will be discussed later in the lecture.

The internal fire prevents the star from collapsing further. When the star is burnt out (i.e. it has little helium left) it may collapse forming something called a white dwarf. Once the sun reaches this state it will be about as large as the earth.

## More massive stars

When a more massive star collapses there will be more fusion reactions leading to an iron core. Once this core becomes so large (and heavy) that it can no longer support itself, it collapses suddenly (in one second) and causes a supernova explosion. For a brief time this star will be as bright as 4 billion suns.

The last supernova in our galaxy was in 1604. Such events show a new star that will only persist for a few days. Some supernovas can be so bright that they are visible by the naked eye during the day (as in 1006)!

# Neutron stars and black holes

If the atoms are pressed close together by gravity they are held apart by a purely quantum-mechanical force the electron exchange force. But if the mass is too big (10-30 suns) this force is not sufficient and the atoms collapse leaving a **neutron star** that consists only of the atomic nuclei densely put together. (Observed by Jocelyn Bell as radio "beeps").

The further collapse is halted by another force the neutron-neutron exchange force. But if the star is even more massive then even that force is not strong enough and the collapse continues. Then a **black hole** is formed. It is so massive that nothing can escape its gravity, not even light! That limiting area where nothing can escape is called the event horizon and is typically 10 to 50 km in radius.

# Newtonian worldview

The Newtonian worldview describes our world as a democratic world, where the same laws of Physics are valid for the whole universe.

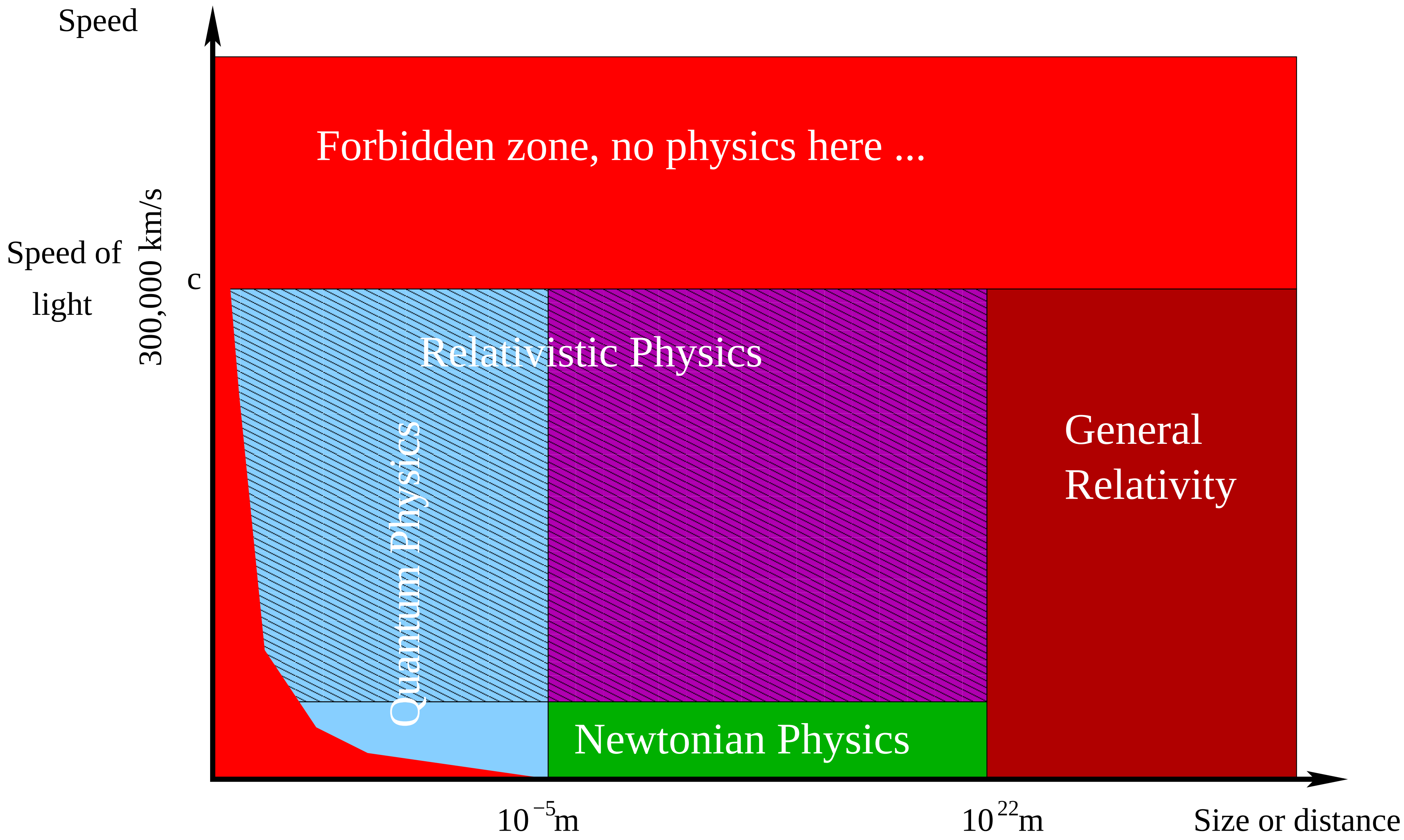
There is no special place for earth, it is just one of many planets in the universe.

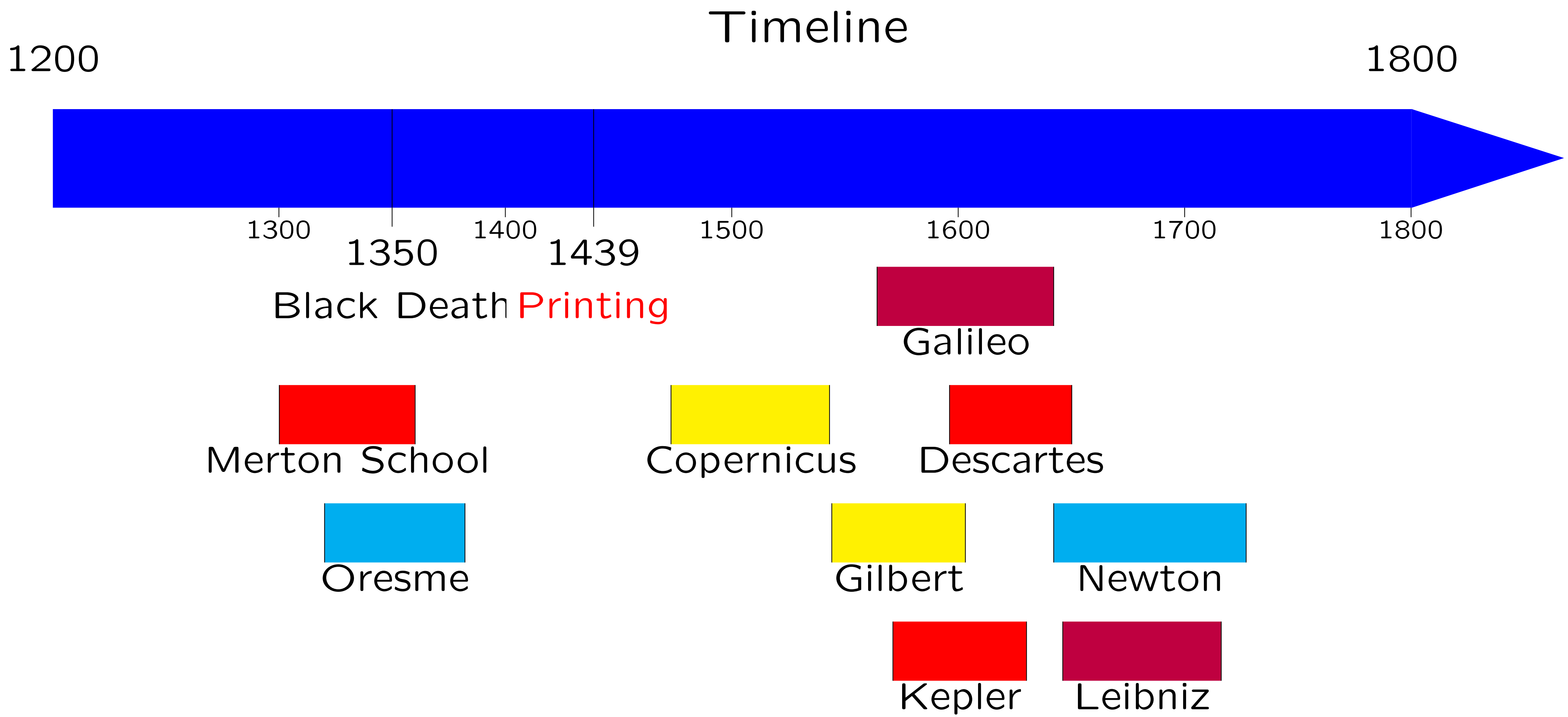
And the universe is mechanical, in the sense that no supernatural being is required to explain the motion of the planets or any other physical phenomenon.

The universe is only a collection of atoms. Thus the future is determined by what all the atoms of the universe are doing at one instance.

Many people felt this was a cold and inhuman worldview, but the success of Newtonian physics led to the wide acceptance of this perspective.

# Limitations of Newtonian Physics




Timeline
1200
1800
1300
1350
1400
1439
1500
1600
1700
1800
Black Death
Printing
Galileo
Merton School
Copernicus
Descartes
Oresme
Gilbert
Newton
Kepler
Leibniz

# Summary

- Newton's theory of Gravity

- Stellar evolution

- Neutron stars and black holes

- The Newtonian worldview

- Limits of validity