Due January 30th Choose 5 problems to solve
1.2.17) Let G_n be the graph whose vertices are permutations of $\{1, \ldots, n\}$, where two permutations are adjacent if they differ by interchanging a pair of adjacent entries (See book for G_3). Prove that G_n is connected.

1.2.18) Let G be the graph whose vertex set is the set of k-tuples with elements in $\{0, 1\}$, and edges between tuples that differ in exactly two positions.
1.2.21) Let G be a self-complementary graph. Prove that G has a cut-vertex if and only if G has a vertex of degree 1.

1.2.22) Prove that a graph is connected if and only if for every partition of its vertices into two non-empty sets, there is an edge with endpoints in both sets.

1.2.24) Let G be a graph having no isolated vertices, and no induced subgraphs with exactly two edges. Prove that G is a complete graph.

1.2.27) Let G_n be the graph who vertices are the permutations of $\{1, \ldots n\}$, with two permutations adjacent if they differ by swapping a pair of entries (not necessarily adjacent entries, as in Problem 7). Prove that G_n is bipartite. (Hint: count the number of pairs $i < j$ with $a_i > a_j$. These are called *inversions.*).

1.2.30) Let G be a simple graph with vertices $v_1, \ldots, v_n$. Let A^k be the kth power of the adjacency matrix A under matrix multiplication. Prove that the i, j entry of A^k is the number of v_i to v_j walks of length k in G. Prove that G is bipartite if and only if, for the nearest odd integer r to n, the diagonal entries of A^r are all 0.

1.2.39) Suppose that every vertex of a loopless graph G has degree at least 3. Prove that G has a cycle of even length. (Hint: consider a maximal path).

1.2.42) Let G be a connected simple graph that does not have C_4 or P_4 as induced subgraphs. Prove that G has a vertex adjacent to all other vertices. (Hint: consider a vertex of maximum degree)

Bonus) Suppose that one knows $(\lambda_1 + \ldots + \lambda_n)$, $(\lambda_1^2 + \ldots + \lambda_n^2)$, $\ldots$, $(\lambda_1^k + \ldots + \lambda_n^k)$, $\ldots$ for arbitrarily many k (though I believe up to $k = n$ suffices). Prove that you can recover what λ_1,λ_2, $\ldots \lambda_n$ are (up to reordering).