MATH 165 (CALCULUS I) TEST 1 REVIEW SHEET (COHEN)

General Comments and Advice: The student should regard this review sheet *only* as a sample of potential test problems, and not an end-all-be-all guide to its content. Anything and everything which we have discussed in class is fair game for the test. The test will cover Sections 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 3.1, and 3.2, but all material included in Chapter 1 is considered essential as well. Don't limit your studying to this sheet; if you feel you don't fully understand a particular topic or technique, then do more problems out of your textbook!

Facts About the Test:

(1) There will be five problems, to be completed in a Bluebook. It is your responsibility to bring the Bluebook.
(2) You may bring to the exam an 8.5"×11" **handwritten** note sheet (one side only), to be handed in with the exam.
(3) **Absolutely no graphing calculators or cell phones.** (Scientific or four-function are ok.)
(4) There will be at least one problem on the exam taken directly from a previous homework assignment.

1. Evaluate the following limits, if they exist:

(a) $\lim_{x\to 6}(3x^{-1}+\sqrt{22-x})$

(b) $\lim_{x\to 2^+}\dfrac{x^2+6x+9}{x+3}$

(c) $\lim_{x\to -1}\dfrac{x^2-2x-3}{x+1}$

(d) $\lim_{x\to 4^-}\dfrac{5}{x-4}$

(e) $\lim_{x\to 1^-}\dfrac{x^2-x+3}{1-x}$

(f) $\lim_{x\to \pi^-}\dfrac{1}{\sin x}$

(g) $\lim_{h\to 0}\dfrac{15h-5h^2}{h}$

(h) $\lim_{x\to -\infty}\dfrac{19}{t^3}-14$

(i) $\lim_{t\to\infty}\dfrac{-t^3+5t-4}{3t^3+25t^2}$

(j) $\lim_{\theta\to\infty}\dfrac{10\sin\theta}{\theta}$

(k) $\lim_{x\to\infty} \dfrac{x^2-5x+4}{x-1}$

(l) $\lim_{\theta\to 0} \dfrac{\sin 7\theta}{\sin 5\theta}$

(m) $\lim_{x\to 4} \dfrac{\sqrt{x}-2}{x-4}$

(n) $\lim_{x\to 9^+} \left(\dfrac{1}{\sqrt{x}-3} - \dfrac{3}{x-9}\right)$

(o) $\lim_{x\to 0} x^6 \cos\dfrac{1}{x}$

(p) $\lim_{x\to 0} \dfrac{\sqrt{t^3+25}\sin t}{t}$

(q) $\lim_{x\to -\infty} \dfrac{-15x^3+52}{1000x+1}$

(r) $\lim_{x\to 4} \dfrac{5}{x-4}$

Also look at: Chapter 2 Review Exercises #11–50.

2. Sketch the graph of a function f which satisfies the following properties:

a. $\lim_{x\to 1^-} f(x) = 1$;

b. $\lim_{x\to 1^+} f(x) = 3$;

c. $\lim_{x\to 2^-} f(x) = \infty$;

d. $\lim_{x\to 3}$ exists but does not equal $f(3)$;

e. $\lim_{x\to\infty} f(x) = -2$; and

f. $\lim_{x\to -\infty} f(x) = \infty$.

3. Suppose f is a function which satisfies the inequality $5 - x^2 \le f(x) \le 5\cos x$ for all real numbers x. Find $\lim_{x\to 0} f(x)$.

4. For $f(x) = 3x^2 - 4x + 7$, find the average rate of change of f between:

(a) $x = 0$ and $x = 2$
(b) $x = -1$ and $x = 4$
(c) $x = -1$ and $x = -1 + h$, where $h \neq 0$
(d) $x = a$ and $x = a + z$, where $z \neq 0$

5. For $f(x) = x^2 + x - 3$, find the slope of the secant line connecting the points:

(a) $(1, f(1))$ and $(3, f(3))$
(b) $(1, f(1))$ and $(2, f(2))$
(c) $(1, f(1))$ and $(1.1, f(1.1))$
(d) $(1, f(1))$ and $(1.01, f(1.01))$

6. State the definition of the derivative.

7. Compute the derivative using the definition:
(a) $f(x) = x^2 + x + 3$

(b) $f(x) = \frac{1}{x}$

(c) $f(x) = \sqrt{x+5}$

Also look at: Section 3.1 #17–20, 29–46.

8. Compute the derivative using any rules you know:
(a) $g(x) = 2x^3 - 3x^2 - 12x + 8 + 2x^{-1} + x^{-5}$

(b) $y = (6x^2 + 5)(4x - 7x^3)$

(c) $f(x) = \sqrt[3]{x} + 16x^{5/2}$

(d) $f(x) = 17x^2 - \frac{5}{x} + \frac{1}{\sqrt{x}}$

9. Let $f(x) = x^3 - 6x^2 - 15x + 1$.
   (1) Find $f'(x)$.
   (2) Find all values of x where the slope of the tangent line to the graph of f at x is horizontal.
   (3) (c) Find the equation of the line tangent to the graph of f at $x = 0$.

10. Find the equation of the tangent line to the graph of $y = \sqrt{x}$ at $x = 25$.

11. Find the points on the graph of $f(x) = 12x - x^3$ where the tangent line is horizontal.

12. Find the values of x where $y = x^3$ and $y = x^2 + 5x$ have parallel tangent lines.

Review 1 Partial Answer Key.

Disclaimer: This key was written quickly and not carefully proofread. Let me know if you spot an error.
1. (a) $\frac{9}{2}$ (use continuity)
(b) 5 (use continuity)
(c) -4 (trick: factor and divide)
(d) $-\infty$
(e) ∞
(f) ∞
(g) 15 (trick: factor and divide)
(h) -14
(i) $-\frac{1}{3}$
(j) 0 (use squeeze theorem)
(k) ∞
(l) $\frac{7}{5}$ (trick: factor out $\sin 7\theta/(7\theta)$ and $5\theta/\sin(5\theta)$)
(m) $\frac{1}{4}$ (trick: multiply by the conjugate)
(n) ∞ (trick: add fractions)
(o) 0 (use squeeze theorem)
(p) 5 (trick: factor out $\sin x/x$)
(q) ∞ (r) limit does not exist (check right vs. left)
3. 5

4. (a) 2
(b) 5
(c) $-10 + 3h$
(d) $6a + 3z - 4$

5. (a) 5
(b) 4
(c) 3.1
(d) 3.01

6. Check your notes!

7. (a) $f'(x) = 2x + 1$ (Note the value of $f'(1)$ is suggested by #4!)
(b) $f'(x) = -\frac{1}{x^2}$
(c) $f'(x) = \frac{1}{2\sqrt{x+5}}$

8. (a) $g'(x) = 6x^2 - 6x - 12 - 2x^{-2} - 5x^{-6}$
(b) $y' = -33x^2 - 210x^4 + 20$
(c) $f'(x) = \frac{1}{3}x^{-2/3} + 40x^{3/2}$
(d) $f'(x) = 34x + \frac{5}{x^2} - \frac{1}{2x^{3/2}}$

9. (a) $f'(x) = 3x^2 - 12x - 15$
(b) $x = -1$ and $x = 5$
(c) $y = -15x + 1$

10. $y - 5 = \frac{1}{10}(x - 25)$

11. $x = -2$ and $x = 2$

12. $x = -1$ and $x = \frac{5}{3}$