

## Final Exam - Calculus II (Fall 2015)

INSTRUCTIONS: Complete each of the following problems in your Bluebook. Points will be awarded for both completeness and clarity of solutions. Partial credit will be awarded for partial solutions. Please recall that **cell phones and graphing calculators are not allowed on this exam.**

1. [8 pts] Evaluate  $\int x^3\sqrt{1-x^2}dx$ .
2. [7 pts] Compute the improper integral  $\int_{-1}^1 \frac{1}{x^5}dx$ , or explicitly demonstrate that it diverges.
3. [8 pts] Let  $R$  be the region between the graph of  $y = 2 \ln x$ , the  $x$ -axis, and the lines  $x = 1$  and  $x = e$ . Find the volume of the solid generated when  $R$  is revolved about the  $y$ -axis.
4. [8 pts] Determine whether the series  $\sum_{n=1}^{\infty} (-1)^n \cdot \frac{8n^{3/2} - \sqrt{n}}{4n^{5/2} - 5n + 1}$  converges absolutely, converges conditionally, or diverges. Completely justify your answer.
5. [8 pts] Determine the interval of convergence of the power series  $\sum_{k=1}^{\infty} \frac{(x+1)^k}{3^k \cdot k^5}$ .
6. [7 pts] Find the 5th degree term in the Maclaurin series expansion for  $x^2 e^{2x}$ .
7. [7 pts] Use the identity  $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$  to find the Maclaurin series representation for  $\cos^2 x$ .
8. [7 pts] Compute the infinite sum  $\sum_{k=2}^{\infty} \frac{5 \cdot (-1)^k}{6^k}$ .

**BONUS. (+5 pts)** Compute the infinite sum  $\frac{\pi}{2} - \frac{\pi^3}{2^3 \cdot 3!} + \frac{\pi^5}{2^5 \cdot 5!} - \frac{\pi^7}{2^7 \cdot 7!} + \dots$