

Final Exam - Calculus II (Spring 2015)

INSTRUCTIONS: Complete each of the following problems in your Bluebook. Points will be awarded for both completeness and clarity of solutions. Partial credit will be awarded for partial solutions. Please recall that **cell phones and graphing calculators are not allowed on this exam.**

1. [7 pts] Determine whether the series $\sum_{n=1}^{\infty} (-1)^n \frac{12n^2 - 5n}{3n^4 - 5n + 1}$ converges absolutely, converges conditionally, or diverges. Completely justify your answer.

2. [8 pts] Evaluate $\int \frac{2x - 1}{x^3 - 5x^2 + 6x} dx$.

3. [8 pts] Use an integral to compute the volume of a sphere of radius 1.

4. [7 pts] Compute $\sum_{n=1}^{\infty} \frac{10 \cdot (-2)^n}{3^n}$, or explain why the series diverges.

5. [8 pts] Evaluate $\int \frac{1}{x^2 \sqrt{x^2 - 36}} dx$.

6. [8 pts] Compute the improper integral $\int_0^{\infty} x e^{-x} dx$.

7. [7 pts] Determine the interval of convergence for the power series $\sum_{k=0}^{\infty} \frac{(x + 10)^k}{k \cdot 4^k}$.

8. [7 pts] Find the third degree term in the Taylor series expansion for $\ln x$ centered at $a = 2$.

BONUS. (+8 pts) Suppose f is a function which admits the Maclaurin series representation

$$f(x) = \sum_{k=0}^{\infty} c_k x^k = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \dots \text{ for } -\infty < x < \infty.$$

Now further suppose that $f'(x) = f(x)$ for every real number x , and that $f(0) = 1$. Determine the value of each Maclaurin coefficient c_k for $k \geq 0$.

(*Hint:* First find c_0 . Then derive the power series for f to find the next few terms c_k for $k \geq 1$. Lastly, deduce the pattern.)