MATH 166 FINAL EXAM STUDY GUIDE

The Final Exam will be comprehensive, but with an emphasis on recent material, namely power series, Taylor series, and Maclaurin series (Sections 10.6 and 10.7). You can also expect problems based on material from earlier in the semester, namely Sections 7.1–7.3, 7.5–7.7, 6.1–6.4, 8.1, and 10.1–10.5. Here is a summary of what to expect on the final and how to prepare:

Content: The exam will be roughly 35% Taylor series (10.6 and 10.7), 25% integration techniques (Chapter 7), 15% applications of the integral (Chapters 6 and 8), and 25% sequences and series (10.1–10.5).

Previous Review Sheets: All the content highlighted in our previous review sheets is still considered relevant.

Previous Tests: The student should make sure she has mastered all the problems of the previous tests, as there will be one or more repeated problems on the final.

Facts About the Final Exam:

(1) There will be approximately eight or nine problems, to be completed in a Bluebook. It is your responsibility to bring the Bluebook.
(2) You may bring to the exam an 8.5”×11” note sheet (both sides!), to be handed in with the exam.
(3) **Absolutely no graphing calculators or cell phones.** (Scientific or four-function are ok.)
(4) There will be at least one problem on the exam taken directly from a previous exam.

Suggested (Two-Sided!) Note Sheet Content:

- Trigonometric identities: Pythagorean, half-angle, double-angle
- Integration by parts formula
- Integration by trigonometric substitution strategy
- Integration by partial fractions strategy
- Washer method
- Shell method
- Arc length of a graph
- Surface area of graph rotated about x-axis
- Criteria for DCT, LCT, Ratio Test, Root Test
- Geometric series formula
- Maclaurin series for: $\frac{1}{1-x}$, $\frac{1}{1+x}$, $\ln(1-x)$, $\ln(1+x)$, e^x, $\sin x$, $\cos x$, $\arctan x$
- Formula for Taylor/Maclaurin coefficients
- Anything basic that you have trouble remembering! Antiderivatives of trig functions, etc.

1. Determine the radius of convergence of the following power series. Then test the endpoints to determine the interval of convergence.

(a) $\displaystyle\sum_{k=0}^{\infty}\left(\frac{x}{3}\right)^k$

(b) $\displaystyle\sum_{k=1}^{\infty}\frac{k^2(x-4)^k}{k!}$

(c) $\displaystyle\sum_{n=1}^{\infty}\frac{x^n}{n^{2/3}}$

(d) $\displaystyle\sum_{k=0}^{\infty}\frac{(-1)^k(x+5)^k}{3^k}$

(e) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^k(x-10)^k}{10^k\sqrt{k}}$

(f) $\displaystyle\sum_{k=1}^{\infty}k^{2k}(x-2)^k$

2. For the functions below, use the formula for Maclaurin coefficients to compute the first four non-zero terms of the Maclaurin series for the given function.

(a) $e^x \sin x$

(b) $e^{\sin x}$

3. Use the Maclaurin series you already know to find the Maclaurin series for the functions below. (Don't worry about interval of convergence.)

(a) $\cos(5x)$

(b) $\ln(1-x^2)$

(c) $x^7 \sin x^4$

(d) $\frac{1}{1+x^2}$

(e) $\arctan x$ *Hint:* Integrate the series you found for part (d).

Also try: Section 10.7 #3, 11, 12, 16, 29, 33, 42, 46

Answers:
1. a. $R = 3$, $IOC = (-3, 3)$ (geometric series)
b. $R = \infty$, $IOC = (-\infty, \infty)$ (Sugg: Ratio Test)
c. $R = 1$, $IOC = [-1, 1)$ (Sugg: Ratio Test, and AST for left endpoint)
d. $R = 3$, $IOC = (-8, -2)$ (Sugg: Root Test)
e. $R = 10$, $IOC = (0, 20]$ (Sugg: Ratio Test, and AST for right endpoint)
f. $R = 0$, converges only at $x = 2$ (Sugg: Root Test)

2. a. $e^x \sin x = x + x^2 + \frac{1}{3}x^3 - \frac{1}{30}x^5 + ...$
b. $e^{\sin x} = 1 + x + \frac{1}{2}x^2 - \frac{1}{8}x^4 + ...$

3. a. $\displaystyle\sum_{k=0}^{\infty}(-1)^k \frac{25^k x^{2k}}{(2k)!}$
b. $\displaystyle -\sum_{k=1}^{\infty} \frac{x^{2k}}{k}$
c. $\displaystyle\sum_{k=0}^{\infty}(-1)^k \frac{x^{8k+11}}{(2k+1)!}$
d. $\displaystyle\sum_{k=0}^{\infty}(-1)^k x^{2k}$
e. $\displaystyle\sum_{k=1}^{\infty}(-1)^k \frac{x^{2k+1}}{2k+1}$