MATH 166 TEST 1 REVIEW SHEET

General Comments and Advice: The student should regard this review sheet *only* as a sample of potential test problems, and not an end-all-be-all guide to its content. Anything and everything which we have discussed in class is fair game for the test. The test will cover Sections 7.1, 7.2, 7.3, 7.5. 7.6, and 7.7, but all material included in Chapter 5 is considered essential as well. Don't limit your studying to this sheet; if you feel you don't fully understand a particular topic or technique, then do more problems out of your textbook!

Facts About the Test:

(a) There will be five problems, to be completed in a Bluebook. It is your responsibility to bring the Bluebook.
(b) You may bring to the exam an 8.5"×11" **handwritten** note sheet (one side only), to be handed in with the exam.
(c) **Absolutely no graphing calculators or cell phones.** (Scientific or four-function are ok.)
(d) There will be at least one problem on the exam taken directly from a previous homework assignment.

1. Evaluate the following integrals. (Be conscious of which integration technique you use and why it's an appropriate choice for the given problem.)

(a) $\displaystyle\int \frac{3}{x-10}dx$

(b) $\displaystyle\int \frac{x^2}{2x^3+1}dx$

(c) $\int \tan x dx$

(d) $\int_0^{\pi/2} \cos x \cdot e^{\sin x} dx$

(e) $\displaystyle\int_1^{e^2} \frac{(\ln x)^5}{x}dx$

(f) $\int_0^3 xe^{4x}dx$

(g) $\int x^2 \ln x dx$

(h) $\int x \sec^2 x dx$

(i) $\int_0^{\pi} e^x \sin x dx$

(j) $\int \sin^2 x dx$

(k) $\int \cos^5 x dx$

(l) $\int \sin^5 x \cos^{-2} x dx$

(m) $\displaystyle\int \frac{1}{(9-x^2)^{3/2}}dx$

(n) $\displaystyle\int_0^9 \frac{1}{\sqrt{x^2+81}}dx$

(o) $\displaystyle\int \frac{1}{x\sqrt{x^2-36}}dx$

(p) $\displaystyle\int_0^1 \frac{1}{(4+9x^2)^2}dx$

(q) $\displaystyle\int \frac{1}{(x-1)(x+2)}dx$

(r) $\displaystyle\int \frac{3}{x^2-x-12}dx$

(s) $\displaystyle\int \frac{\ln x}{x^{10}}dx$

(t) $\displaystyle\int \frac{y+1}{y^3+3y^2-18y}dy$

(u) $\int_1^\infty x^{-2}dx$

(v) $\int_5^\infty \frac{1}{\sqrt{x}}dx$

(w) $\displaystyle\int_0^\infty \frac{x}{\sqrt[5]{x^2+1}}dx$

(x) $\int_0^{32} \frac{1}{\sqrt[5]{x}}dx$

(y) $\displaystyle\int_0^1 \frac{x^3}{x^4-1}dx$

(z) $\int_0^1 \ln x^2 dx$ (*Hint:* Compute $\lim_{a\to 0^+} \dfrac{\ln a}{a^{-1}}$ using L'Hospital's rule.)

(aa) $\displaystyle\int_{-3}^3 \frac{1}{\sqrt{9-x^2}}dx$

(ab) $\int_{-1}^1 \frac{1}{x}dx$

2. Let n be a positive integer. Show that $\int x^n \ln x dx = \frac{1}{n+1}x^{n+1}\ln x - \frac{1}{(n+1)^2}x^{n+1} + C$.

Answer Key

Disclaimer: This key was written quickly and is likely to contain errors or typos! Please let me know if you are sure you have detected one.

1 a. $3\ln|x-10|+C$
b. $\frac{1}{6}\ln|2x^3+1|+C$
c. $\ln|\sec x|+C$
d. $e-1$
e. $\frac{16}{3}$
f. $\dfrac{11e^{12}+1}{16}$
g. $\frac{1}{3}x^3\ln x-\frac{1}{9}x^3+C$
h. $x\tan x-\ln|\sec x|+C$
i. $\frac{1}{2}(e^{\pi}+1)$
j. $\frac{1}{2}x-\frac{1}{4}\sin 2x+C$
k. $\sin x-\frac{2}{3}\sin^3 x+\frac{1}{5}\sin^5 x+C$
l. $\sec x+2\cos x-\frac{1}{3}\cos^3 x+C$
m. $\frac{x}{9\sqrt{9-x^2}}+C$
n. $\ln(\sqrt{2}+1)$
o. $\frac{1}{6}\text{arcsec}\ \frac{x}{6}+C=\frac{1}{6}\arccos\frac{6}{x}+C$
p. $\frac{1}{48}\arctan\frac{3}{2}+\frac{6}{13}$
q. $\frac{1}{3}\ln|x-1|-\frac{1}{3}\ln|x+2|+C$
r. $\frac{3}{7}\ln|x-4|-\frac{3}{7}\ln|x+3|+C$
s. $-\frac{1}{9}x^{-9}\ln x-\frac{1}{81}x^{-9}+C$
t. $-\frac{1}{18}\ln|y|-\frac{5}{54}\ln|y+6|+\frac{4}{27}\ln|y-3|+C$
u. 1
v. diverges
w. diverges
x. 20
y. diverges
z. -2
aa. π
ab. diverges