MATH 166 TEST 2 REVIEW SHEET

General Comments and Advice: The student should regard this review sheet *only* as a sample of potential test problems, and not an end-all-be-all guide to its content. Anything and everything which we have discussed in class is fair game for the test. The test will cover roughly Sections 6.1, 6.2, 6.3, 6.4, and 8.1. Don't limit your studying to this sheet; if you feel you don't fully understand a particular topic, then try to do more problems out of your textbook!

Facts About the Test:

(1) There will be five problems, to be completed in a Bluebook. It is your responsibility to bring the Bluebook.
(2) You may bring to the exam an 8.5"×11" handwritten note sheet (one side only), to be handed in with the exam.
(3) **Absolutely no graphing calculators or cell phones.** (Scientific or four-function are ok.)
(4) There will be at least one problem on the exam taken directly from a previous homework assignment.

1. Find the area of the region bounded by the graphs of $y = x$ and $y = x^3$ between $x = -1$ and $x = 1$.

2. Find the area of the region bounded by $y = \sin x$, $y = \cos x$, and the x-axis between $x = 0$ and $x = \frac{\pi}{2}$.

3. Use an integral to compute the volume of a cone whose height is 12, and whose circular base has radius 5.

4. Let R be the region bounded by the graphs of $y = \sin x$ and $y = 1 - \sin x$ between $x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$.

(a) Find the volume of the solid generated when R is revolved about the x-axis.
(b) Find the volume of the solid generated when R is revolved about the line $y = -3$.

5. Compute the volume of the three-dimensional solid whose base is the region bounded between the graph of $y = 9 - x^2$ and the x-axis, and whose horizontal cross sections (perpendicular to the y-axis) are equilateral triangles.

6. Let R be the region bounded by the graphs of $y = |x|$ and $y = 12 - x^2$. Find the volume of the solid generated when R is revolved about the x-axis.

7. Let R be the region bounded by the graph of $y = 6 - x$, the x-axis, and the lines $x = 2$ and $x = 4$.

(a) Find the volume of the solid generated when R is revolved about the y-axis.
(b) Find the volume of the solid generated when R is revolved about the line $x = 1$.

8. Let R be the same region as in problem 6. Do you get a larger volume by revolving R about the x-axis, or about the y-axis?

9. Find the volume of liquid needed to fill a sphere of radius R to height h.

10. Calculate the arc length over the given interval.
(a) $y = 7x - 3$, $[1, 4]$
(b) $y = \frac{1}{9}x^{3/2} - 3x^{1/2}$, $[2, 8]$

11. Show that the circumference of the unit circle is equal to $2\int_{-1}^{1} \frac{1}{\sqrt{1-x^2}}dx$, an improper integral. Then compute the circumference of the unit circle.

12. Compute the surface area of revolution about the x-axis over the given interval.
(a) $y = x^3$, $[1, 3]$
(b) $y = \frac{1}{4}x^2 - \frac{1}{2}\ln x$, $[1, e]$
(c) $y = e^{-x}$, $[0, 1]$ (*Hint:* Substitute $u = e^{-x}$, and then do a trigonometric substitution.)
(d) $y = \sin x$, $[-\pi, 0]$ (*Hint:* Substitute $u = \cos x$, and then do a trigonometric substitution.)

Answer Key

Disclaimer: This key was written quickly and may contain errors or typos! Please let me know if you have detected one.

1. $\frac{1}{2}$

2. $2-\sqrt{2}$

3. 100π
4. a. $\pi(2\sqrt{3}-\frac{2\pi}{3})$
b. $\pi(14\sqrt{3}-\frac{14\pi}{3}$

5. $\frac{81\sqrt{3}}{4}$
6. $\frac{2556\pi}{5}$

7. a. $\frac{104\pi}{3}$
b. $\frac{68\pi}{3}$

8. About the y-axis, since $\frac{104\pi}{3} > \frac{56\pi}{3}$.

9. $\pi h(1-\frac{1}{3}h^2+hR-R^2)$

10. a. $15\sqrt{2}$
b. $\frac{41}{9}\sqrt{2}$

11. Use the arc length formula to obtain the given expression. The circumference of the unit circle is 2π.

12. a. $\frac{1}{27}\pi(730^{3/2}-10^{3/2})$
b. $\frac{1}{32}e^4+\frac{7}{16}e^2-\frac{7}{32}$
c. $\pi[\sqrt{2}+\ln(\sqrt{2}+1)-e^{-1}\sqrt{1+e^{-2}}+\ln(\sqrt{1+e^{-2}}+e^{-1})]$
d. $2\pi(\sqrt{2}+\ln(\sqrt{2}+1))$