Test 1 - Calculus II (Spring 2015)

INSTRUCTIONS: Complete each of the following problems in your Bluebook. Each problem is worth a maximum of 10 points. Points will be awarded for both completeness and clarity of solutions. Partial credit will be awarded for partial solutions. Please recall that **cell phones and graphing calculators are not allowed on this exam.**

1. Compute $\int \dfrac{12}{x^3 + 2x^2 - 3x} dx$.

2. Compute $\int \sin^3 x \cos^2 x dx$.

3. Compute $\int_0^{5\sqrt{2}/2} \dfrac{25}{(25 - x^2)^{3/2}} dx$.

4. Use integration by parts to show that $\int x^2 \sin x = -x^2 \cos x + 2x \sin x + 2\cos x + C$.

5. Find a value for the constant K which guarantees that $\int_0^\infty \dfrac{K}{(x+4)^3} dx = 1$.

6. Compute $\int_{-3}^3 \dfrac{1}{x^4} dx$, or show that the integral diverges.

BONUS. (+8 points) Use integration by parts to derive the reduction formula for sine:

$$\int \sin^n x dx = -\tfrac{1}{n} \sin^{n-1} x \cos x + \tfrac{n-1}{n} \int \sin^{n-2} x dx,$$

where n may be any integer greater than or equal to 2.

(*Hints:* Set $u = \sin^{n-1} x$. Remember the relationship between $\sin^2 x$ and $\cos^2 x$.)