Test 2 - Calculus II (Fall 2015)

INSTRUCTIONS: Complete each of the following problems in your Bluebook. Each problem is worth a maximum of 12 points. Points will be awarded for both completeness and clarity of solutions. Partial credit will be awarded for partial solutions. Please recall that **cell phones and graphing calculators are not allowed on this exam.**

1. Use an integral to find the volume of a sphere of radius R.

2. The life X (in hours) of a battery in constant use is a random variable with exponential density. What is the probability that the battery will last more than 6 hours if the average life is 8 hours? (*Hint:* Recall that an exponential density function has the form $p(t) = \frac{1}{r}e^{-t/r}$, where r is the average value.)

3. Let R be the region bounded below $y = \sin x$ and above the x-axis, for $0 \leq x \leq \pi$. Find the volume of the solid obtained by revolving R about the line $y = -2$.

4. Find the total area of the region bounded between the graphs of $y = 16x$ and $y = x^5$ for $-2 \leq x \leq 2$.

5. Compute the arc length of the graph of $y = \frac{1}{4}x^3 + \frac{1}{3}x^{-1}$, for $1 \leq x \leq 2$.

BONUS. (+8 points) Let $(a_n)_{n=1}^{\infty}$ be the sequence defined by $a_1 = 0$, and the recurrence relation

$$a_{n+1} = a_n + \frac{1}{(2n-1)(2n+1)}.$$

(a) Find an explicit formula for each term a_n.

(b) Does the sequence converge? If so, to what limit?