Test 3 - Calculus II (Fall 2015)

INSTRUCTIONS: Complete each of the following problems in your Bluebook. Each problem is worth a maximum of 12 points. Points will be awarded for both completeness and clarity of solutions. Partial credit will be awarded for partial solutions. Please recall that **cell phones and graphing calculators are not allowed on this exam.**

1. Determine the convergence or divergence of the following series. Carefully justify your answers.

(a) $\displaystyle\sum_{k=2}^{\infty} \frac{k^{3/2}}{15k \ln k}$ (b) $\displaystyle\sum_{k=1}^{\infty} \frac{k^{20}}{3k^{21} + 15k^7 + 5}$

2. Compute the value of $\displaystyle\sum_{n=4}^{\infty} \frac{2}{(n+4)(n+6)}$, or show that the series diverges.

3. Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n^2}{(2n+1)(2n+2)}\right)^n$ converges absolutely, converges conditionally, or diverges. Carefully justify your answer.

4. Determine whether the series $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1} \ln k}{k}$ converges absolutely, converges conditionally, or diverges. Carefully justify your answer.

5. Determine the interval of convergence of the power series $\displaystyle\sum_{k=1}^{\infty} \frac{(x+1)^k}{3^k \cdot k^5}$.

BONUS. (+8 points) Determine whether $\displaystyle\sum_{n=1}^{\infty} \frac{n^{n-1}}{e^n \cdot n!}$ converges or diverges.

(*Hint:* Use *Stirling's approximation*, which says that the sequences $(n!)$ and $\left(\dfrac{n^n\sqrt{2\pi n}}{e^n}\right)$ have the same growth rate, to find a nice series with which to compare.)