# MONOMIAL IDEALS: HOMEWORK 6

**Exercise 1.** Let $A$ be a commutative ring with identity and let $R$ be the polynomial ring $R = A[X, Y, Z]$ in three variables. Set

$$J = (Z^4, Y^2Z^3, Y^3, XYZ, XY^2, X^2)R.$$

(a) Find the $J$-corner-elements.
(b) Find an irredundant parametric decomposition of $J$.

**Exercise 2.** Show that the conclusion of Corollary III.1.3 does not hold when we drop the assumption that $\operatorname{rad}(I) = \operatorname{rad}((\mathbf{X}))$. (Here is a statement of that corollary: Let $A$ be a commutative ring with identity and let $R$ be the polynomial ring $R = A[X_1, \ldots, X_d]$ in $d$ variables. Set $(\mathbf{X}) = (X_1, \ldots, X_d)R$ and let $I$ and $J$ be monomial ideals in $R$ such that $\operatorname{rad}(J) = \operatorname{rad}((\mathbf{X})) = \operatorname{rad}(I)$. Then $I = J$ if and only if $\mathrm{C}_R(I) = \mathrm{C}_R(J)$.

**Exercise 3.** Let $A$ be a commutative ring with identity and let $R$ be the polynomial ring $R = A[X, Y]$ in two variables. Set $(\mathbf{X}) = (X, Y)R$. Find monomial ideals $I$ and $J$ in $R$ such that $\operatorname{rad}(I) = \operatorname{rad}((\mathbf{X})) = \operatorname{rad}(J)$ and $I \subseteq J$ and $\mathrm{C}_R(I) \cap \mathrm{C}_R(J) = \emptyset$; in particular, you will have $\mathrm{C}_R(I) \not\subseteq \mathrm{C}_R(J)$ and $\mathrm{C}_R(I) \not\supseteq \mathrm{C}_R(J)$.

**Exercise 4.** Let $A$ be a commutative ring with identity and let $R$ be the polynomial ring $R = A[X, Y, Z]$ in three variables. Set

$$I = (X^2, Y, Z)R \cap (X, Y^2, Z)R \cap (X^3, Y, Z^2)R \cap (X, Y^2, Z^3)R \cap (X^2, Y^2, Z^2)R.$$

Find an irredundant parametric decomposition for $I$ and list the $I$-corner elements.