MATH 724, FALL 2009, HOMEWORK 5
DUE FRIDAY 06 NOVEMBER

Exercise 1. (50 pts.) Let R be a commutative ring, and consider R-module homomorphisms $f\colon L \to L'$ and $g\colon M \to M'$ and $h\colon N \to N'$.

(a) Each of the homomorphisms f, g, and h yields a commutative diagram with respect to tensor evaluation. Write out these diagrams.
(b) Each of the homomorphisms f, g, and h yields a commutative diagram with respect to Hom evaluation. Write out these diagrams.

Extra credit: Verify that the diagrams from parts (a) and (b) commute.

Exercise 2. (50 pts.) Let R be a commutative ring, and let E and M be R-modules such that E is injective. Prove the following:

(a) If M is flat, then $\mathrm{Hom}_R(M, E)$ is injective.
(b) If M is injective, then $\mathrm{Hom}_R(M, E)$ is flat.
(c) The converses of (a) and (b) hold when E is faithfully injective.