MATH 724, FALL 2009, HOMEWORK 6
DUE FRIDAY 20 NOVEMBER

Exercise 1. (40 pts.) (Theorem 3.2.1(b)) Let R be a commutative noetherian ring. Let M be an R-module and let C be a semidualizing R-module. Prove that $M \in \mathcal{B}_C(R)$ if and only if $\operatorname{Hom}_R(C, M) \in \mathcal{A}_C(R)$.

Exercise 2. (60 pts.) (Proposition 3.2.8) Let R be a commutative noetherian ring, and let C be a semidualizing R-module. Let F be a flat R-module, and let M be an R-module. Prove the following statements.

  (a) If M is in $\mathcal{A}_C(R)$, then $M \otimes_R F$ is in $\mathcal{A}_C(R)$.
  (b) If M is in $\mathcal{B}_C(R)$, then $M \otimes_R F$ is in $\mathcal{B}_C(R)$.
  (c) The converses of (a) and (b) hold when F is faithfully flat.