MATH 724, FALL 2009, HOMEWORK 7
DUE FRIDAY 04 DECEMBER

Exercise 1. (40 pts.) Let R be a commutative noetherian ring, and let C be a semidualizing R-module. Let F be a flat R-module, and let I be an injective R-module. Prove that the following diagrams commute:

$$\begin{array}{ccccc}
\mathcal{A}_C(R) & \xrightarrow{F\otimes_R -} & \mathcal{A}_C(R) & \xrightarrow{\operatorname{Hom}_R(-,I)} & \mathcal{B}_C(R) \\
{\scriptstyle C\otimes_R -}\big\downarrow & & & & \big\uparrow{\scriptstyle C\otimes_R -} \\
\mathcal{B}_C(R) & \xrightarrow{F\otimes_R -} & \mathcal{B}_C(R) & \xrightarrow{\operatorname{Hom}_R(-,I)} & \mathcal{A}_C(R)
\end{array}$$

$$\begin{array}{ccccc}
\mathcal{A}_C(R) & \xrightarrow{\operatorname{Hom}_R(-,I)} & \mathcal{B}_C(R) & \xrightarrow{F\otimes_R -} & \mathcal{B}_C(R) \\
{\scriptstyle C\otimes_R -}\big\downarrow & & & & \big\uparrow{\scriptstyle C\otimes_R -} \\
\mathcal{B}_C(R) & \xrightarrow{\operatorname{Hom}_R(-,I)} & \mathcal{A}_C(R) & \xrightarrow{F\otimes_R -} & \mathcal{A}_C(R)
\end{array}$$

Exercise 2. (60 pts.) Let R be a commutative noetherian ring, and let C be a semidualizing R-module. Let M and N be R-modules such that $p = \operatorname{pd}_R(M) < \infty$ and $\operatorname{Ext}^i_R(M,N) = 0$ for all $i \neq p$. Prove the following statements.

(a) If $N \in \mathcal{A}_C(R)$, then $\operatorname{Ext}^p_R(M,N) \in \mathcal{A}_C(R)$.
(b) If $N \in \mathcal{B}_C(R)$, then $\operatorname{Ext}^p_R(M,N) \in \mathcal{B}_C(R)$.
(c) The converses of parts (a) and (b) hold when N is finitely generated and $M \cong R/(\mathbf{x})R$ for some R-regular sequence $\mathbf{x}$ in the Jacobson radical of R.