MATH 720, Algebra I
Exercises 1
Due Fri 02 Sep

Exercise 1. Let $\{G_\alpha\}_{\alpha \in A}$ be a set of additive abelian groups, and for each $\lambda \in \Lambda$ let $H_\lambda \leqslant G_\lambda$. Prove that

$$\begin{gathered} \bigoplus_\lambda H_\lambda \leqslant \bigoplus_\lambda G_\lambda \leqslant \prod_\lambda G_\lambda \\ \bigoplus_\lambda H_\lambda \leqslant \prod_\lambda H_\lambda \leqslant \prod_\lambda G_\lambda \\ (\bigoplus_\lambda G_\lambda)/(\bigoplus_\lambda H_\lambda) \cong \bigoplus_\lambda (G_\lambda/H_\lambda) \\ (\prod_\lambda G_\lambda)/(\prod_\lambda H_\lambda) \cong \prod_\lambda (G_\lambda/H_\lambda). \end{gathered}$$

Exercise 2. Deduce that, given additive abelian groups G_1 and G_2, one has

$$\begin{gathered} G_1 \oplus 0 \leqslant G_1 \oplus G_2 \\ 0 \oplus G_2 \leqslant G_1 \oplus G_2 \\ (G_1 \oplus G_2)/(G_1 \oplus 0) \cong G_2 \\ (G_1 \oplus G_2)/(0 \oplus G_2) \cong G_1. \end{gathered}$$

Exercise 3. Prove or give a counterexample: given additive abelian groups G_1 and G_2, every subgroup of $G_1 \oplus G_2$ is of the form $H_1 \oplus H_2$ for some $H_1 \leqslant G_1$ and $H_2 \leqslant G_2$.