MATH 790, FALL 2011, HOMEWORK 2
DUE FRIDAY 16 SEPTEMBER

Exercise 1. Let R be a commutative noetherian ring, and let M be a finitely generated R-module. Prove that M has a free resolution $F_\bullet$ such that each F_i is finitely generated.

Exercise 2. Let R be a commutative noetherian ring, and let $M_\bullet$ be an R-complex. Fix an integer i. Prove that if M_i is finitely generated over R, then $H_i(M_\bullet)$ is finitely generated over R.

Exercise 3. Let G be a finitely generated $\mathbb{Z}$-module, and let H be a $\mathbb{Z}$-module. Prove that $\operatorname{Ext}^i_{\mathbb{Z}}(G, H) = 0$ for all $i > 1$.