MATH 790, FALL 2011, HOMEWORK 5
DUE FRIDAY 07 OCTOBER

Let $\varphi\colon R \to S$ be a flat homomorphism of noetherian rings. (That is, R and S are noetherian, and φ is a ring homomorphism making S into a flat R-module.) Let $U \subseteq R$ be a multiplicatively closed subset.

Exercise 1. Let M and M' be R-modules.

(a) Prove that for each i there is an S-module isomorphism

$$\operatorname{Tor}_i^S(S \otimes_R M, S \otimes_R M') \cong S \otimes_R \operatorname{Tor}_i^R(M, M').$$

(b) Prove that for each i there is $U^{-1}R$-module isomorphism

$$\operatorname{Tor}_i^{U^{-1}R}(U^{-1}M, U^{-1}M') \cong U^{-1} \operatorname{Tor}_i^R(M, M').$$

(c) Prove that there is a $U^{-1}R$-module isomorphism

$$(U^{-1}M) \otimes_{U^{-1}R} (U^{-1}M') \cong U^{-1}(M \otimes_R M').$$

Exercise 2. Let M and C be R-modules.

(a) Prove that there is a commutative diagram of S-module homomorphisms

$$\begin{array}{ccc} S \otimes_R M & \xrightarrow{\delta^{S\otimes_R C}_{S\otimes_R M}} & \operatorname{Hom}_S(\operatorname{Hom}_S(S \otimes_R M, S \otimes_R C), S \otimes_R C) \\ \Big\downarrow{\scriptstyle S\otimes_R \delta^C_M} & & \Big\downarrow \\ S \otimes_R \operatorname{Hom}_R(\operatorname{Hom}_R(M, C), C) & \longrightarrow & \operatorname{Hom}_S(S \otimes_R \operatorname{Hom}_R(M, C), S \otimes_R C) \end{array}$$

(b) Prove that there is a commutative diagram of $U^{-1}R$-module homomorphisms

$$\begin{array}{ccc} U^{-1}M & \xrightarrow{\delta^{U^{-1}C}_{U^{-1}M}} & \operatorname{Hom}_{U^{-1}R}(\operatorname{Hom}_{U^{-1}R}(U^{-1}M, U^{-1}C), U^{-1}C) \\ \Big\downarrow{\scriptstyle U^{-1}\delta^C_M} & & \Big\downarrow \\ U^{-1} \operatorname{Hom}_R(\operatorname{Hom}_R(M, C), C) & \longrightarrow & \operatorname{Hom}_{U^{-1}R}(U^{-1} \operatorname{Hom}_R(M, C), U^{-1}C) \end{array}$$