MATH 720, Algebra I
Exercises 4
Due Wed 01 Oct

Exercise 1. Set $R = \mathbb{Z}[x]$. Prove that the ideal $\langle 2, x\rangle$ is not principal.

Exercise 2. Prove Proposition 2.2.7(d) from the notes.

Exercise 3. Let R be a commutative ring with identity, and let I, J be ideals of R. Prove that $I \bigcup J$ is an ideal of R if and only if either $I \subseteq J$ or $J \subseteq I$.

Exercise 4. (a) Prove Proposition 2.2.11(c) from the notes.

(b) Let A be a commutative ring with identity. Consider the polynomial ring $R = A[x_1, \ldots, x_d]$ and the ideal $\langle x_1, \ldots, x_d\rangle \subseteq R$. Prove that for each $n \in \mathbb{N}_+$, the ideal $\langle x_1, \ldots, x_d\rangle^n$ is generated by the set $\{x_1^{a_1} \cdots x_d^{a_d} \mid \sum_i a_i = n\}$.