Math 165
Spring 2009
Midterm 2 practice exam answers.

1. Find the instantaneous rate of change of the area of the square with respect to side-length when the side-length is 10 cm.
Answer: $20\,\text{cm}^2/\text{cm}$

2. A population of tribbles is initially 20 and grows at a rate that is proportional to its size. After 10 days, the population has increased to 30.
(a) How many tribbles will there be after 47 days?
Answer: $20\exp(\frac{47}{10}\ln(1.5))$ tribbles
(b) How long will it take for the population to reach 100?
Answer: $10\ln(5)/\ln(1.5)$ days

3. Starting with a right triangle with sides 3,4,5 cm we increase the small side at a rate of 2 cm/min, keeping the side of length 4 constant. How fast is the smallest angle changing when it reaches the value of $\pi/4$ radians?
Answer: 1/4 radians/min

4. Use a linear approximation to estimate $\sqrt{4.01}$.
Answer: 2.0025

5. Find the absolute maximum value for the function $f(x) = x^3 - 3x + 1$ on the interval $[0, 2]$.
Answer: 3

6. Show that the equation $x^7 + 2x^3 + 1 = 0$ has a solution in the interval $[-2, 1]$
Answer: The function $f(x) = x^7 + 2x^3 + 1$ is differentiable. Since $f(-2)$ is negative and $f(1)$ is positive, the Intermediate Value Theorem shows that the equation $f(x) = 0$ has a solution in the interval $(-2, 1)$.

7. Consider the function $f(x) = \dfrac{2x^2 - 1}{x^2 + 1}$.

(a) Find the critical values of f.

Answer: $x = 0$

(b) Find the intervals where f is increasing and where f is decreasing.

Answer: decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$.

(c) Find the local maxima and local minima for f.

Answer: The only local min is -1 at $x = 0$. There is no local max.

(d) Find the intervals where the graph of f is concave upward and where it is concave downward.

Answer: concave up on $(-\sqrt{3}/3, \sqrt{3}/3)$, and concave down on $(-\infty, -\sqrt{3}/3)$ and $(\sqrt{3}/3, \infty)$.

(e) Find the inflection points for f.

Answer: $(-\sqrt{3}/3, -1/4)$ and $(\sqrt{3}/3, -1/4)$

(f) Find the asymptotes for the graph of f.

Answer: horizontal asymptote at $y = 2$ and no vertical asymptote.

(g) Find the x-intercepts and the y-intercepts for the graph of f.

Answer: The x-intercepts are $(-\sqrt{2}/2, 0)$ and $(\sqrt{2}/2, 0)$, and the y-intercept is $(0, -1)$.

(h) Sketch the graph of f indicating the relevant data from (a)–(g).

Answer: see extra page.

Bonus. Find numbers b and c so that the point $(1, 0)$ is an inflection point for the function $x^3 + bx^2 + cx + 1$.

Answer: $b = -3$ and $c = 1$.