MATH 499/696, SPRING 2010, HOMEWORK 5
DUE FRIDAY 09 APRIL

Item numbers refer to version of course notes dated March 29, 2010

Exercise 3.5.26: Let A be a commutative ring with identity, and let R be the polynomial ring $R = A[X_1, \ldots, X_d]$ in d variables. Let I and J be monomial ideals in R.

(a) Prove that $\text{m-rad}\,(I) \subseteq \text{m-rad}\,(J)$ if and only if $\text{rad}(I) \subseteq \text{rad}(J)$.

(b) Prove that $\text{m-rad}\,(I) = \text{m-rad}\,(J)$ if and only if $\text{rad}(I) = \text{rad}(J)$.

Exercise 3.5.27: Let A be a commutative ring with identity and let R be the polynomial ring $R = A[X_1, \ldots, X_d]$ in d variables. Set $(\mathbf{X}) = (X_1, \ldots, X_d)R$. Prove that, if I is a monomial ideal, then $\text{m-rad}\,(I) = (\mathbf{X})$ if and only if $\text{rad}(I) = \text{rad}((\mathbf{X}))$.

Exercise 3.5.32: Let A be a commutative ring with identity and let R be the polynomial ring $R = A[X_1, \ldots, X_d]$ in d variables. Let $I_1, \ldots, I_n$ be monomial ideals in R and set $(\mathbf{X}) = (X_1, \ldots, X_d)R$.

(a) Prove that, if $\text{m-rad}\,(I_j) = (\mathbf{X})$ for $j = 1, \ldots, n$, then $\text{m-rad}\,(I_1 \cdots I_n) = (\mathbf{X})$.

(b) Prove that if $I_j \neq R$ for $j = 1, \ldots, n$ and $\text{m-rad}\,(I_1 \cdots I_n) = (\mathbf{X})$, then $\text{m-rad}\,(I_j) = (\mathbf{X})$ for $j = 1, \ldots, n$.

Exercise 3.5.36: Let A be a commutative ring with identity and let R be the polynomial ring $R = A[X_1, \ldots, X_d]$ in d variables. Set $(\mathbf{X}) = (X_1, \ldots, X_d)R$, and let I and J be monomial ideals of R. Prove that if $I \subseteq (\mathbf{X})$ and $\text{m-rad}\,(J) = (\mathbf{X})$, then $(J :_R I) \supsetneq J$.

Exercise 3.5.31: [Math 696 only] Let A be a commutative ring with identity and let R be the polynomial ring $R = A[X_1, \ldots, X_d]$ in d variables. Let $(\mathbf{X}) = (X_1, \ldots, X_d)R$. Let I be a monomial ideal in R, and let S denote the set of monomials in $R \smallsetminus I$. Prove that S is a finite set if and only if $\text{m-rad}\,(I) \supseteq (\mathbf{X})$.