# MATH 724, SPRING 2012, HOMEWORK 2
# DUE WEDNESDAY 15 FEBRUARY

**Exercise 1.** Assume that $k$ is algebraically closed. Let $f$ be a non-constant polynomial in $k[X_1, \dots, X_n]$, and consider $V(f) \subset \mathbb{A}^n_k$.
(a) Prove that $V(f) \neq \emptyset$.
(b) Prove that if $n \geqslant 2$, then $V(f)$ is infinite.

**Exercise 2.** Are the following closed sets irreducible or not? Justify your responses.
(a) $V(X + Y^2) \subseteq \mathbb{A}^2_{\mathbb{R}}$.
(b) $V(X^2 + Y^2) \subseteq \mathbb{A}^2_{\mathbb{R}}$.
(c) $V(X^2 + Y^2) \subseteq \mathbb{A}^2_{\mathbb{C}}$.

**Exercise 3.** For $i = 0, 1, \dots, n$ set

$$U_i = \{(v_0 : v_1 : \dots : v_n) \in \mathbb{P}^n_k \mid v_i \neq 0\} = \{(w_0 : w_1 : \dots : w_n) \in \mathbb{P}^n_k \mid w_i = 1\}.$$

(a) Prove that the map $f_i \colon \mathbb{A}^n_k \to U_i$ given by

$$f_i(x_1, \dots, x_n) = (x_1 : \dots : x_i : 1 : x_{i+1} : \dots : x_n)$$

is a well-defined bijection.
(b) Prove that the map $g_i \colon \mathbb{P}^{n-1}_k \to \mathbb{P}^n_k \smallsetminus U_i$ given by

$$g_i(x_0 : \dots : x_{n-1}) = (x_0 : \dots : x_{i-1} : 0 : x_i : \dots : x_{n-1})$$

is a well-defined bijection.

**Exercise 4.** Let $X$ be a noetherian topological space. Let $Y \subseteq X$ be a subspace of $X$, that is, a subset of $Y$ with the subspace topology. Prove that $Y$ is noetherian.