MATH 724, SPRING 2012, HOMEWORK 3
DUE WEDNESDAY 29 FEBRUARY

Exercise 1. Prove that if k is an infinite field, then $\dim(\mathbb{A}^n_k) = n = \dim(\mathbb{P}^n_k)$.

Exercise 2. Let $0 \neq f, g \in k[X_0, X_1, \ldots, X_n]_d$ where $d \geqslant 1$. And consider $U_g \subseteq \mathbb{P}^n_k$.

(a) Prove that the function $q \colon U_g \to k$ given by $q(\mathbf{a}) = f(\mathbf{a})/g(\mathbf{a})$ is well-defined.

(b) Use this idea to explicitly construct the inverse to the function f_i from Homework 2, Exercise 3(a).

Exercise 3. Identify $\mathbb{A}^{n^2}_k$ with the set of $n \times n$ matrices in a natural way, and prove that the set of invertible $n \times n$ matrices is an open subset of $\mathbb{A}^{n^2}_k$.

Exercise 4. Let k be a finite field with p^t elements. What is the cardinality of $\mathbb{P}^n_k$? Justify your answer.