MATH 726, SPRING 2012, HOMEWORK 11
DUE FRIDAY 20 APRIL

Let R be a commutative ring with identity.

Exercise 1. Let M be an R-module, and let $g\colon N \to N'$ be an R-module isomorphism. Prove that $\operatorname{Ext}^i_R(M, g)$ and $\operatorname{Ext}^i_R(g, M)$ are isomorphisms. (The same is true for $\operatorname{Tor}^R_i(M, g)$ and $\operatorname{Tor}^R_i(g, M)$, but you do not need to prove it.)

Exercise 2. Let M and N be R-modules.

(a) Let $r \in R$. Prove that if $rM = 0$ or $rN = 0$, then $r \operatorname{Ext}^i_R(M, N) = 0$ for all i.

(b) Prove that $\operatorname{Ann}_R(M) \cup \operatorname{Ann}_R(N) \subseteq \operatorname{Ann}_R(\operatorname{Ext}^i_R(M, N))$ for all i, and conclude that $\operatorname{Ann}_R(M) + \operatorname{Ann}_R(N) \subseteq \cap_{i=0}^{\infty} \operatorname{Ann}_R(\operatorname{Ext}^i_R(M, N))$

(The same is true for $\operatorname{Tor}^R_i(M, N)$, but you do not need to prove it.)