MATH 724, SPRING 2012, HOMEWORK 2
DUE FRIDAY 03 FEBRUARY

Exercise 1. Let R be a commutative ring with identity, and let $U \subseteq R$ be a multiplicatively closed subset. Let M be an R-module, and let $h\colon N \to N'$ be an R-module homomorphism. Prove that the following diagram commutes:

$$\begin{array}{ccc}
U^{-1}\operatorname{Hom}_R(M,N) & \xrightarrow{\Theta_{U,M,N}} & \operatorname{Hom}_{U^{-1}R}(U^{-1}M,U^{-1}N) \\
{\scriptstyle U^{-1}\operatorname{Hom}_R(M,h)}\downarrow & & \downarrow{\scriptstyle \operatorname{Hom}_{U^{-1}R}(U^{-1}M,U^{-1}h)} \\
U^{-1}\operatorname{Hom}_R(M,N') & \xrightarrow{\Theta_{U,M,N'}} & \operatorname{Hom}_{U^{-1}R}(U^{-1}M,U^{-1}N').
\end{array}$$

Exercise 2. Let R be a commutative ring with identity, and let $U \subseteq R$ be a multiplicatively closed subset. Let M, M', and N be R-modules, and prove that the following diagram commutes.

$$\begin{array}{ccc}
U^{-1}\operatorname{Hom}_R\left(\begin{smallmatrix} M \\ \oplus \\ M' \end{smallmatrix}, N\right) & \xrightarrow{\Theta_{U,M\oplus M',N}} & \operatorname{Hom}_{U^{-1}R}\left(U^{-1}\begin{pmatrix} M \\ \oplus \\ M' \end{pmatrix}, U^{-1}N\right) \\
\downarrow{\scriptstyle \cong} & & \downarrow{\scriptstyle \cong} \\
U^{-1}\begin{pmatrix} \operatorname{Hom}_R(M,N) \\ \oplus \\ \operatorname{Hom}_R(M',N) \end{pmatrix} & & \operatorname{Hom}_{U^{-1}R}\begin{pmatrix} U^{-1}M & \\ \oplus & ,U^{-1}N \\ U^{-1}M' & \end{pmatrix} \\
\downarrow{\scriptstyle \cong} & & \downarrow{\scriptstyle \cong} \\
\begin{matrix} U^{-1}\operatorname{Hom}_R(M,N) \\ \oplus \\ U^{-1}\operatorname{Hom}_R(M',N) \end{matrix} & \xrightarrow{\begin{matrix}\Theta_{U,M,N} \\ \oplus \\ \Theta_{U,M',N}\end{matrix}} & \begin{matrix} \operatorname{Hom}_{U^{-1}R}(U^{-1}M,U^{-1}N) \\ \oplus \\ \operatorname{Hom}_{U^{-1}R}(U^{-1}M',U^{-1}N) \end{matrix}.
\end{array}$$

Here, the unlabeled vertical isomorphisms are (induced by) the natural ones from Exercises I.3.3(c) and I.4.24 in the notes. (Another version of the diagram is on the next page.)

$$
\begin{array}{ccc}
U^{-1}\operatorname{Hom}_R(M \oplus M', N) & \xrightarrow{\Theta_{U,M\oplus M',N}} & \operatorname{Hom}_{U^{-1}R}(U^{-1}(M \oplus M'), U^{-1}N) \\
\downarrow \cong & & \downarrow \cong \\
U^{-1}(\operatorname{Hom}_R(M,N) \oplus \operatorname{Hom}_R(M',N)) & & \operatorname{Hom}_{U^{-1}R}(U^{-1}M \oplus U^{-1}M', U^{-1}N) \\
\downarrow \cong & & \downarrow \cong \\
U^{-1}\operatorname{Hom}_R(M,N) \oplus U^{-1}\operatorname{Hom}_R(M',N) & \xrightarrow{\Theta_{U,M,N}\oplus\Theta_{U,M',N}} & \operatorname{Hom}_{U^{-1}R}(U^{-1}M, U^{-1}N) \oplus \operatorname{Hom}_{U^{-1}R}(U^{-1}M', U^{-1}N).
\end{array}
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