MATH 726, SPRING 2012, HOMEWORK 3
DUE FRIDAY 10 FEBRUARY

Let R and S be commutative rings with identity.

Exercise 1. Let $F\colon \text{Mod-}R \to \text{Mod-}S$ be a covariant functor such that $F(0) = 0$. Prove that the following conditions are equivalent:

(i) F is exact.
(ii) F is left-exact and right-exact.
(iii) F is left-exact and for each R-module epimorphism f the map $F(f)$ is an epimorphism.
(iv) F is right-exact and for each R-module monomorphism f the map $F(f)$ is an monomorphism.

Exercise 2. State, but do not prove, the version of Exercise 1 for a contravariant functor.

Exercise 3. Prove the following result that implies that the tensor product of two R-modules is uniquely determined up to isomorphism: Let M and N be R-modules. Let T and T' be R-modules equipped with R-bilinear maps $h\colon M \times N \to T$ and $h'\colon M \times N \to T'$ satisfying the following universal mapping property: For every R-module L and every R-bilinear map $f\colon M \times N \to L$ there are unique R-module homomorphisms $F\colon T \to L$ and $F'\colon T' \to L$ such that $F \circ h = f = F' \circ h'$. Then there is a unique R-module isomorphism $g\colon T \xrightarrow{\cong} T'$ such that $g \circ h = h'$.