MATH 726, SPRING 2012, HOMEWORK 5
DUE FRIDAY 24 FEBRUARY

Let R be a commutative ring with identity.

Exercise 1. Let $\{N_i\}_{i\in I}$ be a set of R-modules.
(a) Prove that $\coprod_{i\in I} N_i$ is flat if and only if N_i is flat for all $i \in I$.
(b) Prove that if M is a free R-module, then M is flat.
(c) Prove that if M is a projective R-module, then M is flat.

Note that the converses to (b) and (c) fail because $\mathbb{Q}$ is a flat $\mathbb{Z}$-module that is not projective (hence not free).

Exercise 2. Let R be a commutative ring with identity. Let M be an R-module, and let $I \subseteq R$ be an ideal. Prove that $(R/I) \otimes_R M \cong M/IM$. (See the hint in Exercise II.4.14 of the notes.)

Exercise 3. Let $\varphi\colon R \to S$ be a homomorphism of commutative rings with identity, and let M be an R-module.
(a) Prove that if M is injective over R, then $\operatorname{Hom}_R(S, M)$ is injective over S.
(b) Prove that if M is projective over R, then $S \otimes_R M$ is projective over S.

Hint: Feel free to use the exercises from Section II.5 of the notes.