MATH 726, SPRING 2012, HOMEWORK 6
DUE FRIDAY 02 MARCH

Exercise 1. Let R be a commutative ring, and let M be a finitely presented R-module. Prove that the following conditions are equivalent:

(i) M is a projective R-module;
(ii) the localization $U^{-1}M$ is a projective $U^{-1}R$-module for each multiplicatively closed subset $U \subseteq R$;
(iii) the localization $M_{\mathfrak{p}}$ is a projective $R_{\mathfrak{p}}$-module for each prime ideal $\mathfrak{p} \subset R$; and
(iv) the localization $M_{\mathfrak{m}}$ is a projective $R_{\mathfrak{m}}$-module for each maximal ideal $\mathfrak{m} \subset R$.

Extra Credit: Provide an example showing that this fails if M is not finitely presented.

Exercise 2. Let R be a commutative ring that is an integral domain. Prove that the following conditions are equivalent:

(i) The ring R is a field;
(ii) Every R-module is free;
(iii) Every R-module is projective;
(iv) Every R-module is injective.

Provide an example showing that this fails if R is not an integral domain.