MATH 726, SPRING 2012, HOMEWORK 8
DUE FRIDAY 23 MARCH

Exercise 1. Let R be a commutative ring. Let F be a flat R-module, and let $\phi\colon A \to B$ and $\psi\colon B \to C$ be R-module homomorphisms.

(a) Prove that there are R-module isomorphisms $F \otimes_R \operatorname{Im}(\phi) \cong \operatorname{Im}(F \otimes_R \phi)$ and $(F \otimes_R B)/\operatorname{Im}(F \otimes_R \phi) \cong F \otimes_R (B/\operatorname{Im}(\phi))$.

(b) Prove that there are R-module isomorphisms $F \otimes_R \operatorname{Ker}(\psi) \cong \operatorname{Ker}(F \otimes_R \psi)$ and $(F \otimes_R B)/\operatorname{Ker}(F \otimes_R \psi) \cong F \otimes_R (B/\operatorname{Ker}(\psi))$.

(c) Assume that $\psi\phi = 0$, and prove that $F \otimes_R \operatorname{Im}(\phi)$ is naturally isomorphic to a submodule of $F \otimes_R \operatorname{Ker}(\psi)$ in such a way that $(F \otimes_R \operatorname{Ker}(\psi))/(F \otimes_R \operatorname{Im}(\phi)) \cong F \otimes_R (\operatorname{Ker}(\psi)/\operatorname{Im}(\phi))$.

Exercise 2. Let R be a commutative ring, and let $\{M_\lambda\}_{\lambda \in \Lambda}$ be a set of R-modules.

(a) Prove that if each module M_λ is flat and some M_μ is faithfully flat, then the coproduct $\coprod_\lambda M_\lambda$ is faithfully flat.

(b) Does the converse of part (a) hold? Justify your answer.

(c) Assume that R is noetherian. State and prove the versions of parts (a) and (b) for the product $\prod_\lambda M_\lambda$.

(d) Assume that R is noetherian, and let I be a set. Prove that R^I is faithfully flat over R.