MATH 726, SPRING 2012, HOMEWORKS 9–10
DUE FRIDAY 13 APRIL

Exercise 1. Let R be a commutative ring, and let $\{M^\lambda\}_{\lambda\in\Lambda}$ be a set of R-modules. For each $\lambda \in \Lambda$, let $P_\bullet^\lambda$ be a projective resolution of M^λ, and let $I_\bullet^\lambda$ be an injective resolution of M^λ.

(a) Prove that $\coprod_{\lambda\in\Lambda} P_\bullet^\lambda$ is a projective resolution of $\coprod_{\lambda\in\Lambda} M^\lambda$.
(b) Prove that if each $P_\bullet^\lambda$ be a free resolution of M^λ, then $\coprod_{\lambda\in\Lambda} P_\bullet^\lambda$ is a free resolution of $\coprod_{\lambda\in\Lambda} M^\lambda$.
(c) Prove that $\prod_{\lambda\in\Lambda} I_\bullet^\lambda$ is an injective resolution of $\prod_{\lambda\in\Lambda} M^\lambda$.
(d) Prove that if R is noetherian, then $\coprod_{\lambda\in\Lambda} I_\bullet^\lambda$ is an injective resolution of the coproduct $\coprod_{\lambda\in\Lambda} M^\lambda$.

Exercise 2. Let R be a commutative ring, let N be an R-module, and let $\{M^\lambda\}_{\lambda\in\Lambda}$ be a set of R-modules.

(a) Prove that $\operatorname{Ext}^i_R(N, \prod_{\lambda\in\Lambda} M^\lambda) \cong \prod_{\lambda\in\Lambda} \operatorname{Ext}^i_R(N, M^\lambda)$ for each i.
(b) Prove that $\operatorname{Ext}^i_R(\coprod_{\lambda\in\Lambda} M^\lambda, N) \cong \prod_{\lambda\in\Lambda} \operatorname{Ext}^i_R(M^\lambda, N)$ for each i.
(c) Prove that if R is noetherian and N is finitely generated, then there is an isomorphism $\operatorname{Ext}^i_R(N, \coprod_{\lambda\in\Lambda} M^\lambda) \cong \coprod_{\lambda\in\Lambda} \operatorname{Ext}^i_R(N, M^\lambda)$ for each i.