CHAPTER 1

SECTION 1.1 – LINEARITY

Definitions
- Total Change
- Average Rate of Change
- Linearly Related
- Linear Model

Concepts/Theorems
- Working with Total Change & Average Rate of Change
- Identifying a Linear Model

SECTION 1.2 – GEOMETRIC & ALGEBRAIC PROPERTIES OF A LINE

Definitions
- Slope of a Line
- Graph of an Equation
- Slope-Intercept Form
- Point-Slope Form

Concepts/Theorems
- Calculating Slope
- Graphing Lines
- Writing the Equation of a Line
- Parallel vs. Perpendicular Lines

SECTION 1.3 – LINEAR MODELS

Definitions
- Scatterplot
- Line of Best Fit
- Linear Regression
- Correlation Coefficient

Concepts/Theorems
- Creating Scatterplots
- Finding Lines of Best Fit and Correlation Coefficients

SECTION 1.4 – GRAPHING EQUATIONS

Definitions
- Intercepts (x-intercept & y-intercept)
- Symmetry (with respect to the x-axis, y-axis, & origin)

Concepts/Theorems
- Point Plotting
- Finding Intercepts Algebraically
- Identifying Symmetry
- Using Calculator to Find Intercepts and Solve Inequalities

SECTION 1.1 – LINEARITY

Definition (Total Change)

What that means:

Algebraically

Geometrically

Example 1
At the beginning of the year, the price of gas was \$4.05 per gallon. At the end of the year, the price of gas was \$3.19 per gallon. What is the total change in the price of gas?

Algebraically	Geometrically

Example 2
Sally was 3’7” tall on the first day of school and 4’2” tall on the last day of school. What is the total change in Sally’s height?

Algebraically	Geometrically

Definition (Average Rate of Change)

What that means:

Algebraically:

Geometrically:

Example 3

John collects marbles. After one year, he had 60 marbles and after 5 years, he had 140 marbles. What is the average rate of change of marbles with respect to time?

Algebraically	**Geometrically**

Definition (Linearly Related)

SECTION 1.2 – GEOMETRIC & ALGEBRAIC PROPERTIES OF A LINE

Definition (Slope)

What that means:

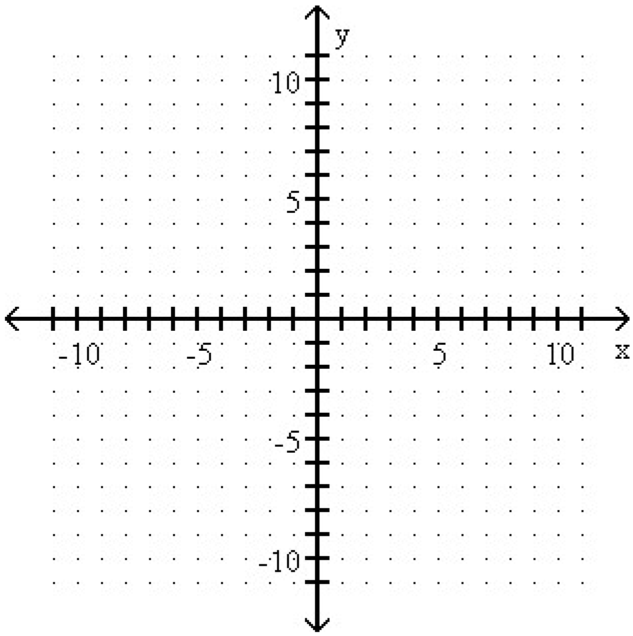
Algebraically

Geometrically

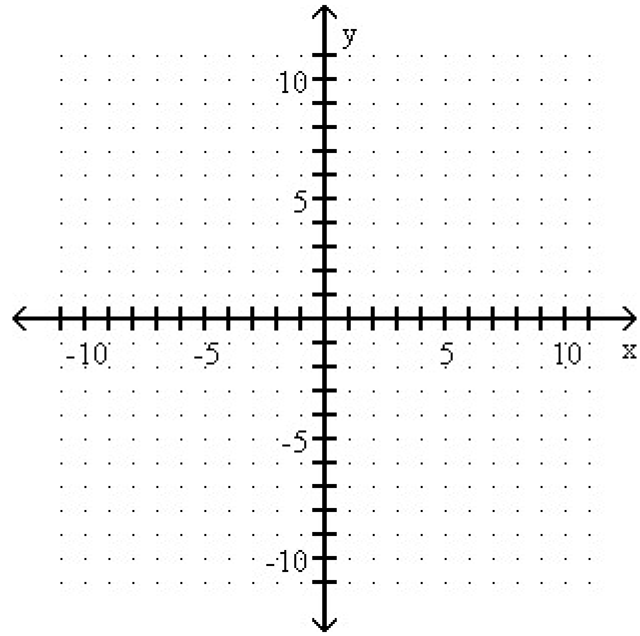
Definition (Graph of an Equation)

Example 1
Graph the line containing the point P with slope m.

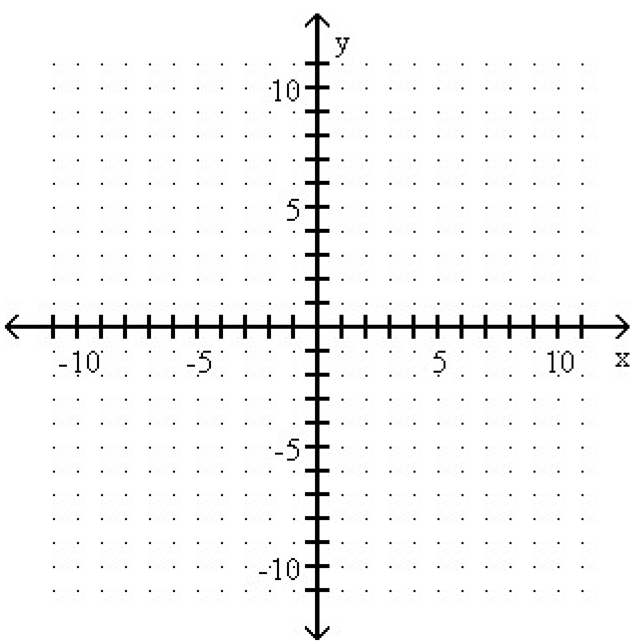
a.) $P = (1,\ 1)$ & $m = -3$



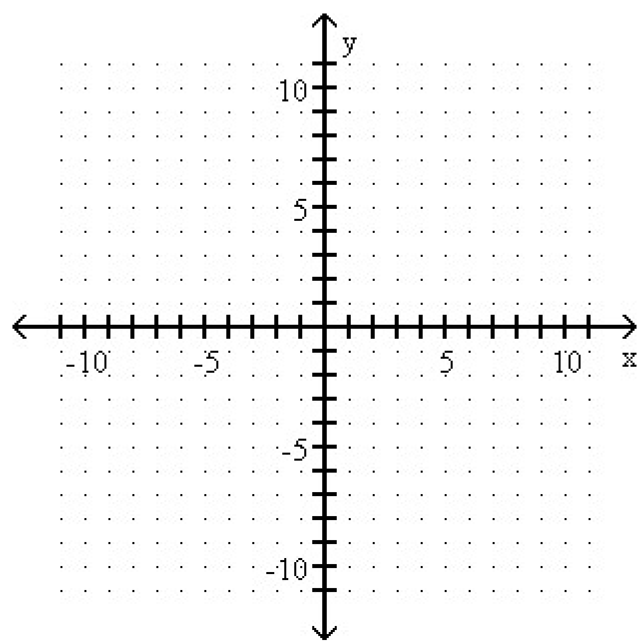
c.) $P = (-1,\ -3)$ & $m = \frac{3}{5}$



b.) $P = (2,\ 1)$ & $m = \frac{1}{2}$



d.) $P = (0,\ -2)$ & $m = -\frac{2}{3}$



Definition (Slope-Intercept Form)

Definition (Point-Slope Form)

Example 2

Find the equation of the line that goes through the points P and Q.

a.) $P = (1,\ 1)$ & $Q = (9,\ 12)$

Using Point-Slope Form	**Using Slope-Intercept Form**

b.) $P = (-2,\ -5)$ & $Q = (3,\ 2)$

Using Point-Slope Form	**Using Slope-Intercept Form**

c.) $P = (-1,\ 2)$ & $Q = (6,\ -4)$

Using Point-Slope Form	**Using Slope-Intercept Form**

d.) $P = (0,\ 6)$ & $Q = (-3,\ -2)$

Using Point-Slope Form	**Using Slope-Intercept Form**

Question

Does it matter which points on the line we use to determine the slope of the line?

Parallel vs. Perpendicular Lines

	Geometrically	**Algebraically**
Parallel Lines		
Perpendicular Lines		

Example 3

a.) Find the equation of a line that is parallel to the line $y = -\frac{6}{7}x - 19$.

b.) Find the equation of a line that is perpendicular to the line $y = -\frac{6}{7}x - 19$.

c.) Find the equation of the line that is parallel to the line $y = -\frac{6}{7}x - 19$ and goes through the point $(2,\ -1)$.

d.) Find the equation of the line that is perpendicular to the line $y = -\frac{6}{7}x - 19$ and goes through the point $(2,\ -1)$.

1.3 LINEAR MODELS

Definition (Scatterplot, Line of Best Fit, Linear Regression, Correlation Coefficient)

- Given a set of data points, the scatterplot is the picture attained by plotting all points.
- The line of best fit (also called the least squares regression line) is the line whose graph best fits the given data.
- Linear regression is the process of finding the line of best fit.
- The correlation coefficient, r, is a measure of how strong the linear relationship is between two quantities. If r is close to zero, there is little to no linear relationship. If $|r|$ is close to one, the quantities are strongly linearly related and the sign of r corresponds to the sign of the slope of the line of best fit.

You will not be tested or quizzed on this section. We are covering it only as a means of gaining experience on the calculator.

Example 1
The following table, from the U.S. Department of Commerce, shows the poverty level for a family of four in selected years (families whose income is below this level are considered to be in poverty.

Year	1990	1991	1992	1993	1994	1995	1996
Income	$13,359	$13,942	$14,335	$14,763	$15,141	$15,569	$16,036

Create a scatterplot and find the line of best fit. Graph the line of best fit on top of your scatterplot. Find the correlation coefficient.

Example 2
The total number of farm workers (in millions) in selected years is shown in the following table.

Year	1900	1920	1930	1940	1950	1960	1970	1980	1990
Workers	29.030	42.206	48.686	51.742	59.230	67.990	79.802	105.06	117.490

Create a scatterplot and find the line of best fit. Graph the line of best fit on top of your scatterplot. Find the correlation coefficient.

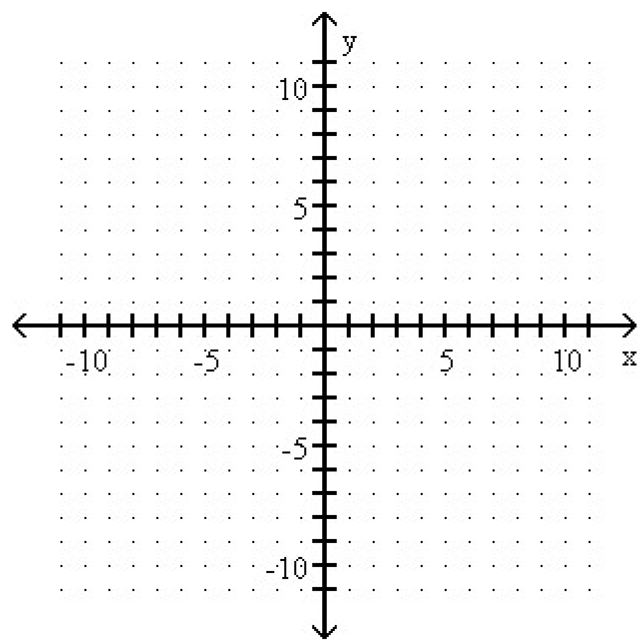
SECTION 1.4 – GRAPHING EQUATIONS

Method of Graphing: Point Plotting

Note:

Example 1

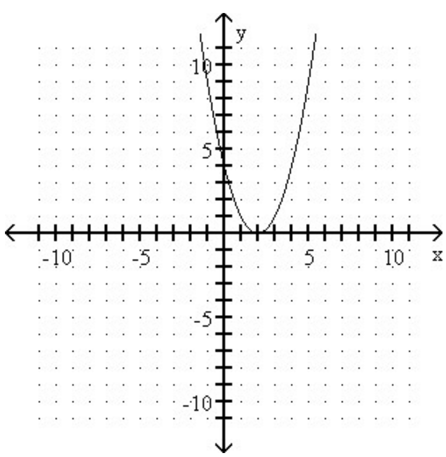
Draw the graph of $y = x^2 - 5$ by point plotting.

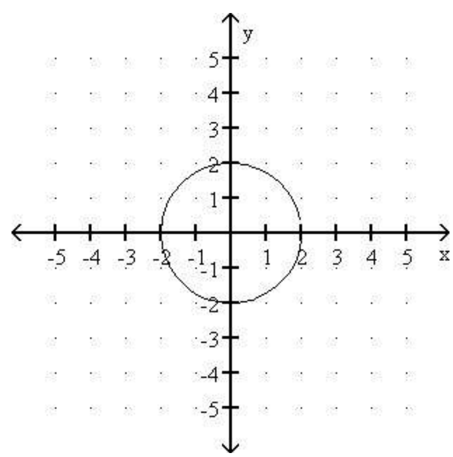


Definition (Intercepts)

Notation

Example 2
Find the intercepts of each of the following graphs.





Finding Intercepts Algebraically

Example 3

Find all intercepts of the following equations.

a.) $x^2 + y^2 = 10$

b.) $y = 2x^2 + 6$

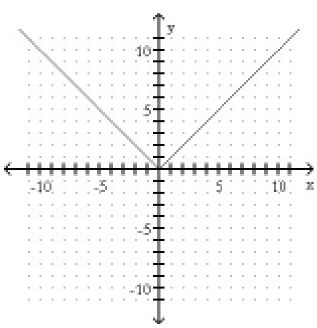
c.) $y = x^3$

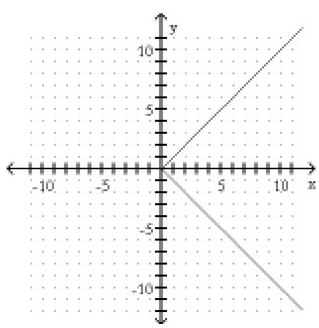
d.) $y = 3 - 3x^3$

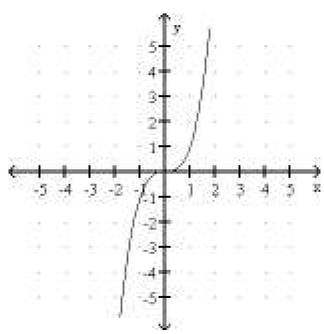
Definition (Symmetry)

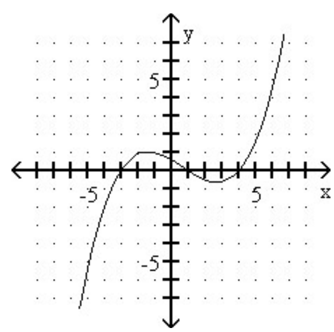
Note:

Example 4
Identify any form of symmetry in each of the following graphs.









Finding Symmetry Algebraically

Note:

Example 5

Determine if each of the graphs of the following equations has symmetry and if so what kind.

a.) $x^2 + y^2 = 10$

b.) $y = 2x^2 + 6$

c.) $y = x^3$

d.) $y = 3 - 3x^3$

Strategy For Solving Inequalities

1.) Move everything to one side of the inequality.

2.) Set $y =$ to the side of the inequality that is not zero.

3.) Find all x-intercepts of your equation.

4.) Use a number line or graph do solve the inequality.

Example 6

Find all values of x such that $2x^2 + 4x + 1 \geq 1$.

CHAPTER 2

SECTION 2.1 – FUNCTIONS FROM A NUMERICAL AND ALGEBRAIC VIEW

Definitions
- Function
- Domain
- Range
- Independent Variable
- Dependent Variable
- Total Change
- Average Rate of Change
- Difference Quotient

Concepts/Theorems
- Describing and Evaluating Functions
- Finding Domain and Range
- Calculating Total Change and Average Rate of Change

SECTION 2.2 – FUNCTIONS FROM A GRAPHICAL VIEW

Definitions
- Monotonicity (Increasing, Decreasing, Constant Behavior)
- Intercepts (using function notation)

Concepts/Theorems
- Interpreting Graphs of Functions
- Vertical Line Test
- Identifying Monotonicity

SECTION 2.3 – MORE MODELING AND PROBLEM SOLVING USING GRAPHS

Concepts/Theorems
- Solving Word Problems

SECTION 2.4 – NEW FUNCTIONS FROM OLD

Definitions
- Sum, Difference, Product, and Quotient Functions
- Function Composition
- Piecewise Defined Function
- Vertical and Horizontal Translations
- Vertical Scalings
- Reflections

Concepts/Theorems
- Creating New Functions From Old Functions

SECTION 2.5: MORE ON DESCRIBING CHANGE

Definitions
- Direct Proportionality
- Inverse Proportionality
- Joint Proportionality
- Combined Proportionality
- Difference Quotient

Concepts/Theorems
- Solving Proportionality Problems
- Revisiting Average Rate of Change and Difference Quotient

SECTION 2.1 – FUNCTIONS FROM A NUMERICAL AND ALGEBRAIC VIEW

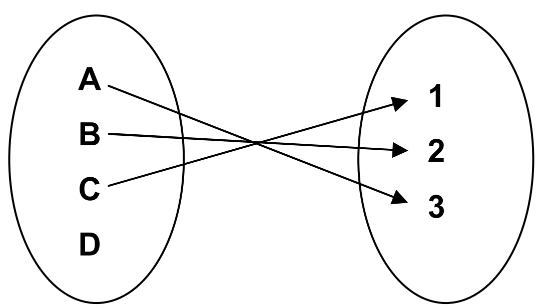
Definition (Function, Domain, & Range)

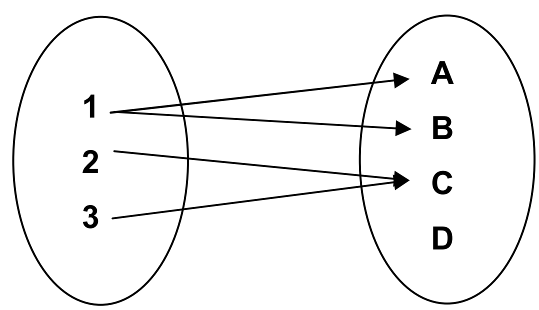
Example 1

Voting for President can be thought of as a function. Assuming that everyone who can vote does vote, then the domain is the set of American citizens who are age 18 or older and the range is the set of all Presidential nominees who receive at least one vote. Notice that each voter can only vote for one candidate. That is, for each input there is exactly one output.

Example 2

Determine if each of these is a function. Also, identify the inputs, outputs, domain, and range where applicable.





Example 3
The following table displays the number of students registered in each section of underwater basket weaving.

Section # (S)	1	2	3	4	5	6	7	8	9	10
# of Students Enrolled (E)	25	32	19	22	41	16	32	22	27	31

Is E a function of S?	
What are the inputs?	
What are the outputs?	
What is the domain?	
What is the range?	
Is S a function of E?	

Definition (Independent vs. Dependent Variables)

Function Notation

Evaluating a Function

Example 4

Let $f(x) = 4x^2 + 3x + 2$. Find each of the following.

$f(0) =$

$f(2) =$

$f(\frac{1}{2}) =$

$f(2k+5) =$

$f(\_) =$

$f(\Delta) =$

$\dfrac{f(t+h) - f(t)}{h} =$

Finding Domain

Hint:

Watch for zero in denominators and negatives under even radicals.

Example 5

Find the domain of each of the following functions.

$f(x) = 2x + 4$

$g(x) = \sqrt[6]{2x - 3}$

$h(x) = \dfrac{4x + 3}{x^2 - 1}$

$k(x) = \dfrac{\sqrt{1 + 6x}}{2x^2 - 3x - 2}$

Implicitly Defined Functions

Example 6
Determine if each of the following equations describes y as a function of x, x as a function of y, or both.

$4x + 3y = 2$

$4x^2 - 3y = 6$

$2x + 4 = y^2$

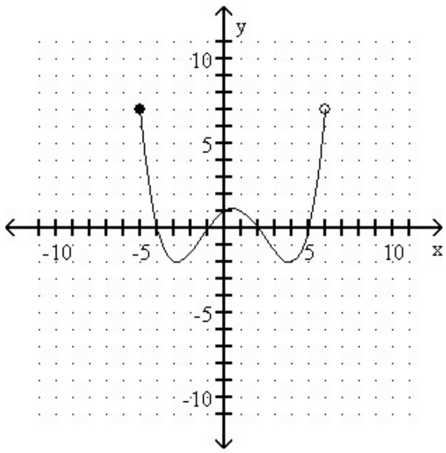
Definition (Total Change, Average Rate of Change, & Difference Quotient)

Example 7
A ball is thrown straight up into the air. The height of the ball in feet above the ground is described by the function $h(t) = -2t^2 + 14$ where time is measured in seconds. Find the total change and average rate of change in height after 2 seconds and interpret each.

SECTION 2.2 – FUNCTIONS FROM A GRAPHICAL VIEW

Example 1

f(x)



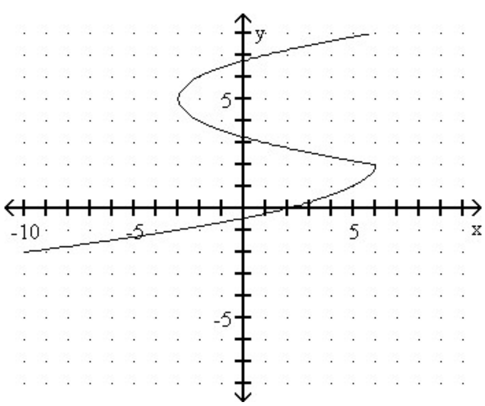
Domain:

Range:

$f(4) =$

$f(-4) =$

Intercepts:

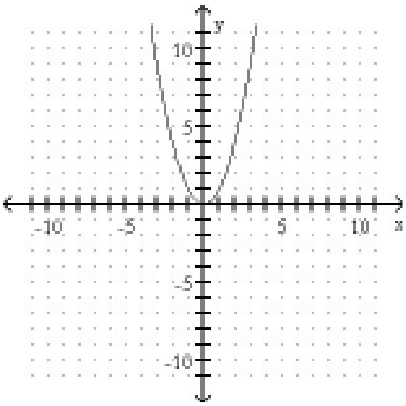


Explain why this graph does not represent a function.

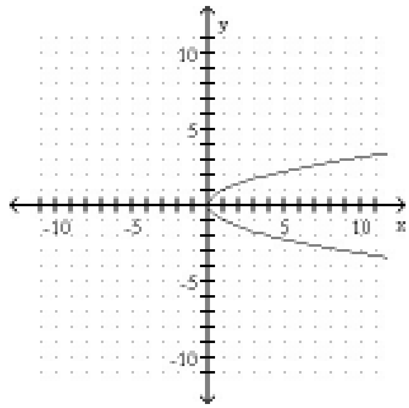
Theorem (Vertical Line Test)

Example 2
Use the vertical line test to determine if each of these describes y as a function of x.

$y = x^2$



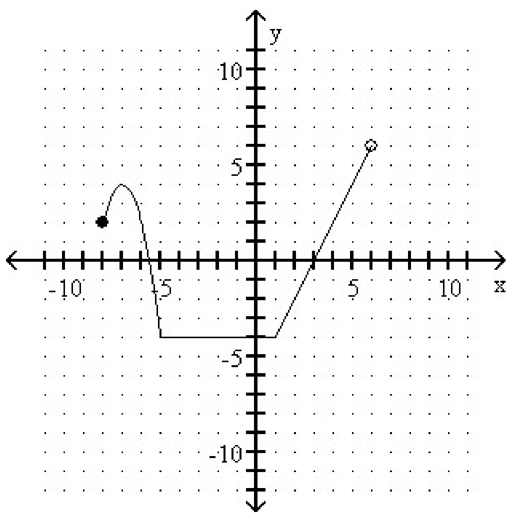
$y^2 = x$



Definition (Monotonicity, Increasing, Decreasing, Constant Behavior)

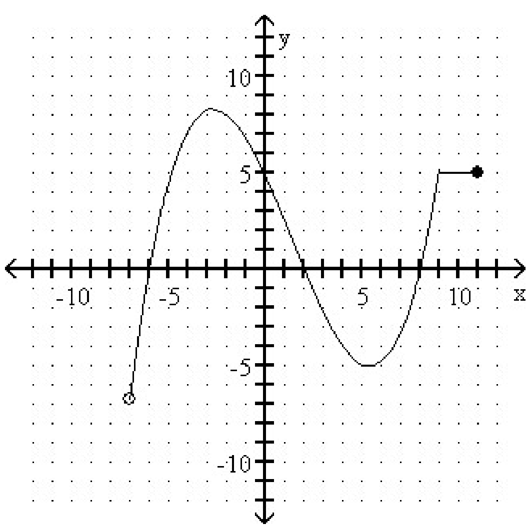
Definition (Intercepts)

Example 3



Domain:	
Range:	
y-intercept:	
x-intercepts:	
Increasing:	
Decreasing:	
Constant:	

Example 4



Domain:	
Range:	
y-intercept:	
x-intercepts:	
Increasing:	
Decreasing:	
Constant:	

SECTION 2.3 – MORE MODELING AND PROBLEM SOLVING USING GRAPHS

Strategy For Solving Story Problems

1.) Read the problem carefully.
2.) Draw a picture (if possible).
3.) Write down everything that is known and expand on this as much as possible.
4.) Write down what you are looking for.
5.) Write an equation that relates what is know to what you are looking for.
6.) Simplify the equation so that it contains only two variables.
7.) Solve.

Example 1

A homeowner has $320 to spend on building a fence around a rectangular garden. Three sides of the fence will be constructed with wire fencing at a cost of $2 per linear foot. In order to provide a view block for a neighbor, the fourth side is to be constructed with wood fencing at a cost of $6 per linear foot. Find the dimensions and the area of the largest garden that can be enclosed with $320 worth of fencing.

Cost, Revenue, Price-Demand, & Profit Functions

- $\text{Cost Function} = (\text{fixed costs}) + (\text{variable costs})$
- $\text{Revenue Function} = (\#\ \text{of items sold})(\text{price per item})$
- Price-Demand Function is a function based on past data and is used in profit-loss analysis. (price per item is determined by the number of items sold)
- $\text{Profit Function} = \text{Revenue} - \text{Cost}$

If $\text{Revenue} < \text{Cost}$, then the company experiences a loss. *Negative Profit.*	If $\text{Revenue} = \text{Cost}$, the company will break even. *No Profit and No Loss.*	If $\text{Revenue} > \text{Cost}$, the company is making money. *Positive Profit.*

Example 2

When a management training company prices its seminar on management techniques at $400 per person, 1000 people will attend the seminar. For each $5 reduction in the price, the company estimates an additional 20 people will attend the seminar. How much should the company charge for the seminar in order to maximize its revenue? What is the maximum revenue?

SECTION 2.4 – NEW FUNCTIONS FROM OLD

Definition (Sum, Difference, Product, & Quotient Functions)

Example 1

Let $f(x) = 2x + 4$ and $g(x) = \sqrt{x+5}$. Find each of the following.

$(f + g)(x) =$

$(f - g)(x) =$

$(g - f)(2) =$

$(fg)(x) =$

$\left(\frac{f}{g}\right)(x) =$

$\left(\frac{g}{f}\right)(-2) =$

Definition (Composite Functions)

Note:

Example 2

Let $f(x) = 2x + 4$ and $g(x) = \sqrt{x+5}$. Find each of the following.

$(f \circ g)(x) =$

$(g \circ f)(x) =$

$(f \circ f)(2) =$

$(f \circ g \circ f)(1) =$

$(g \circ f \circ f)(x) =$

Theorem

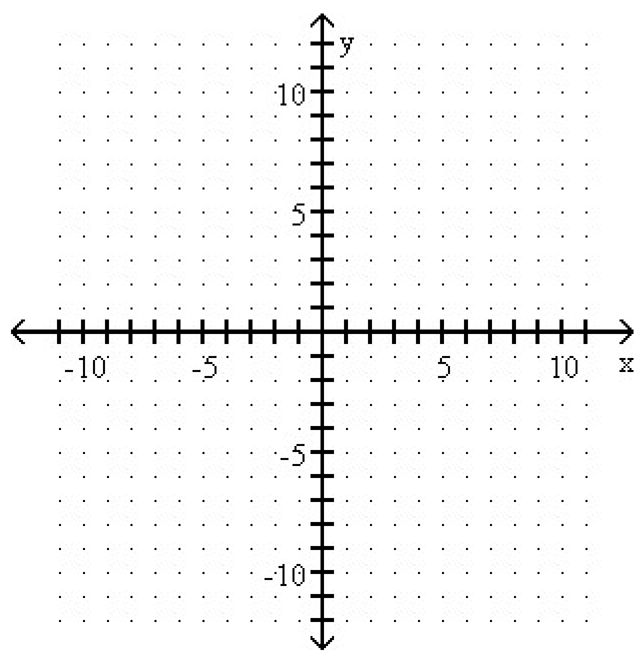
Definition (Piecewise Defined Function)

Example 3
In May 2006, Commonwealth Edison Company supplied electricity to residences for a monthly customer charge of \$7.58 plus 8.275¢ per kilowatt-hour (KWhr) for the first 400 KWhr supplied in the month and 6.208¢ per KWhr for all usage over 400 KWhr in the month. If C is the monthly charge for x KWhr, express C as a function of x.

Example 4

The function f is defined by $f(x)=\begin{cases}-x+1 & \text{if} \quad -1\le x<1 \\ 2 & \text{if} \quad x=1 \\ x^2 & \text{if} \quad x>1\end{cases}$.

a.) Find $f(0)$, $f(1)$, and $f(2)$.
b.) Determine the domain of f.
c.) Graph f.
d.) Use the graph to determine the range of f.



Transformations of Functions

Definition (Translations)

Example 5
Let $f(x) = x^3$. The graph of the function $g(x)$ is the graph of $f(x)$ shifted left 3 units and up 1 unit. Write the formula for $g(x)$.

Definition (Vertical Stretch/Compression)

Example 6

Let $f(x)=|x|$. The graph of the function $g(x)$ is the graph of $f(x)$ stretched by a factor of 2, shifted down 4 units and shifted right 5 units. Write the formula for $g(x)$.

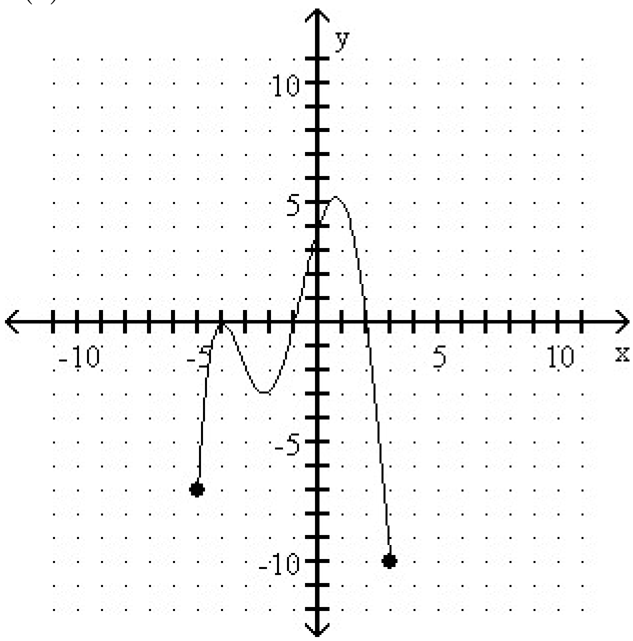
Definition (Reflections)

Example 7

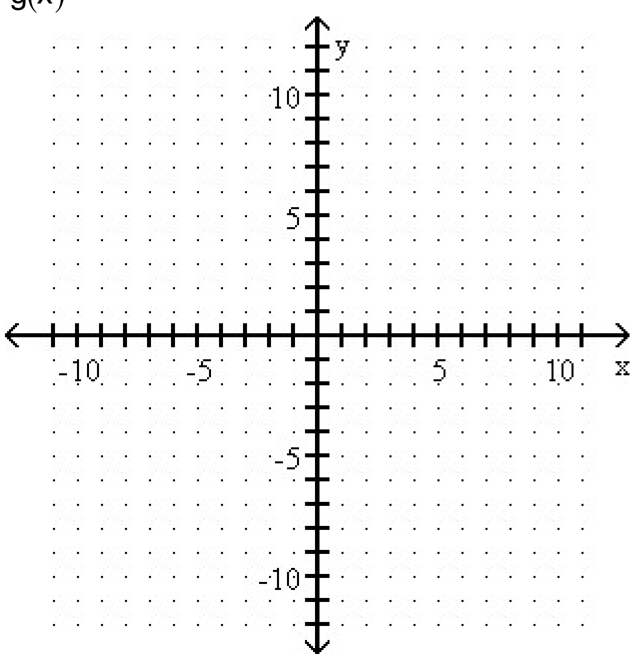
Let $f(x)=\sqrt{x}$. The graph of the function $g(x)$ is the graph of $f(x)$ shifted up 2 units, compressed by a factor of $1/2$, and reflected across the y-axis. Write the formula for $g(x)$.

<u>Example 8</u>
The following is a graph of $f(x)$. Use this graph to draw the graph of $g(x) = -2f(x+2) - 3$.

$f(x)$



$g(x)$



SECTION 2.5 : MORE ON DESCRIBING CHANGE

Definition (Direct Proportionality)

Example 1
The number of centimeters, W, of water produced from melting snow is directly proportional to the number of centimeters, S, of snow. Meteorologists know that under certain conditions, 150cm of snow will melt to produce 16.8cm of water. The average annual snowfall in Alta, Utah is 500cm. How much water will be produced if all of the snow in Alta melts?

Definition (Inverse Proportionality)

Example 2
The ultraviolet, or UV, index is a measure issued daily that indicates the strength of the sun's rays in a particular location. For those people whose skin is quite sensitive, a UV rating of 7 will cause sunburn after 10 minutes. Given that the number of minutes it takes to burn, t, is inversely proportional to the UV rating, u, how long will it take a highly sensitive person to burn on a day with a UV rating of 2?

Definition (Joint Proportionality)

Example 3
The tension, T, on a string in a musical instrument is jointly proportional to the string's mass per unit length, m, the square of its length, L, and the square of its fundamental frequency, f. A 2m long string of mass 5gm/m with a fundamental frequency of 80 has a tension of 100N. How long should the same string be if its tension is going to be changed to 72N?

Definition (Combined Proportionality)

Example 4
The volume, V, of a given mass of gas is directly proportional to the temperature, T, and inversely proportional to the pressure, P. If the volume is 231 cm^3 when the temperature is 42° and the pressure is $20kg/cm^2$, what is the volume when the temperature is 30° and the pressure is $15kg/cm^2$?

Definition (Difference Quotient)

Example 5

If a ball is thrown into the air with a velocity of 40ft/s, its height in feet after t seconds is given by $y = 40t - 16t^2$. Find the average velocity of the ball for the 2 seconds immediately after the ball reaches its maximum height.

CHAPTER 3

SECTION 3.1 – QUADRATIC FUNCTIONS

Definitions
- Quadratic Function
- Extreme Point & Extreme Value

Concepts/Theorems
- Standard Form vs. Special Algebraic Form
- Characteristics of Quadratic Function Graphs
- Solving Inequalities

SECTION 3.2 – POLYNOMIAL FUNCTIONS OF HIGHER DEGREE

Definitions
- Degree, Coefficients, Leading Term, & Leading Coefficient
- Power Function
- Limiting Behavior
- Continuous
- Turning Point

Concepts/Theorems
- Characteristics of Power Function Graphs
- Identifying Limiting Behavior & Number of Turning Points

SECTION 3.3 – POLYNOMIALS AND THEIR DIVISION PROPERTIES

Definitions
- Divisor, Dividend, Quotient, Remainder
- Zero of a Function
- Multiplicity

Concepts/Theorems
- Long Division vs. Synthetic Division
- Remainder Theorem
- Factor Theorem
- Even-Odd Rule

SECTION 3.4 – RATIONAL FUNCTIONS

Definitions
- Rational Function
- Asymptotes
- Hole

Concepts/Theorems
- Characteristics of $R(x) = \left(\frac{1}{x}\right)^n$ Graphs
- Theorem 1 & 2 – Finding Asymptotes

SECTION 3.1 – QUADRATIC FUNCTIONS

Definition (Quadratic Function)

Note:

Completing the Square

1.) Subtract c from both sides.

2.) Divide both sides by a.

3.) Add $\left(\frac{b}{2a}\right)^2$ to both sides.

4.) Factor the right side as $\left(x+\frac{b}{2a}\right)^2$.

5.) Solve for $f(x)$.

Example 1

Convert $f(x)=ax^2+bx+c$ into special algebraic form.

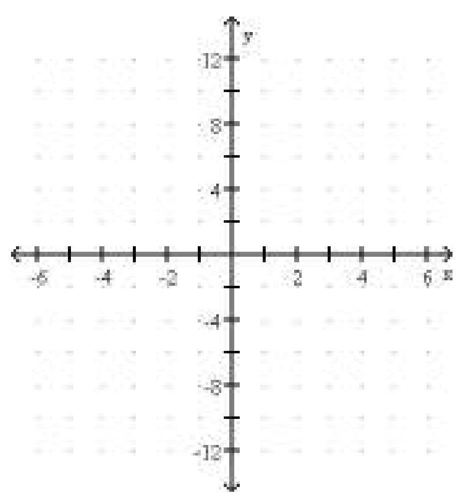
Example 2

Convert $f(x)=2x^2-3x+4$ into special algebraic form.

Definition (Extreme Point & Extreme Value)

Example 3

Draw the graph of $f(x)=2\left(x-\frac{3}{4}\right)^2+\frac{23}{8}$.



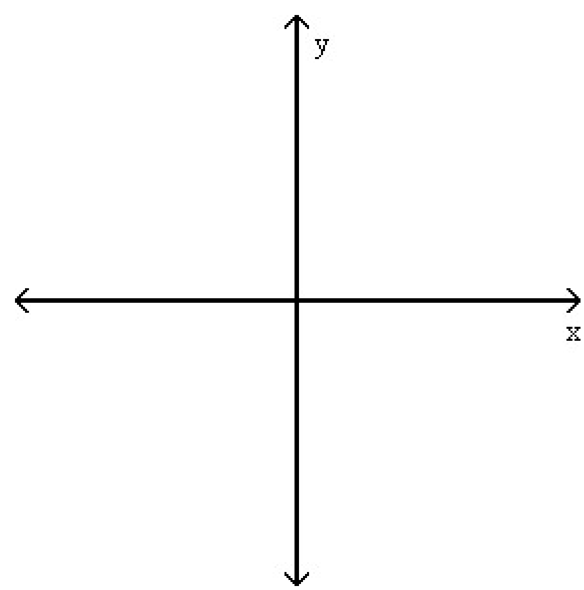
Characteristics of the Graph of $f(x) = a(x-h)^2 + k$						
	Parabola Opens	**Vertex**	**Axis of Symmetry**	**Extreme Point**	**Extreme Value**	**Illustration**
$a > 0$						
$a < 0$						

<u>Solving Inequalities Involving Quadratics</u>

1.) Move everything to one side and set up a quadratic function.
2.) Find the x-intercepts of this function.
3.) Determine if the parabola opens up or down.
4.) Use a rough sketch of the graph to solve the inequality.

<u>Example 4</u>

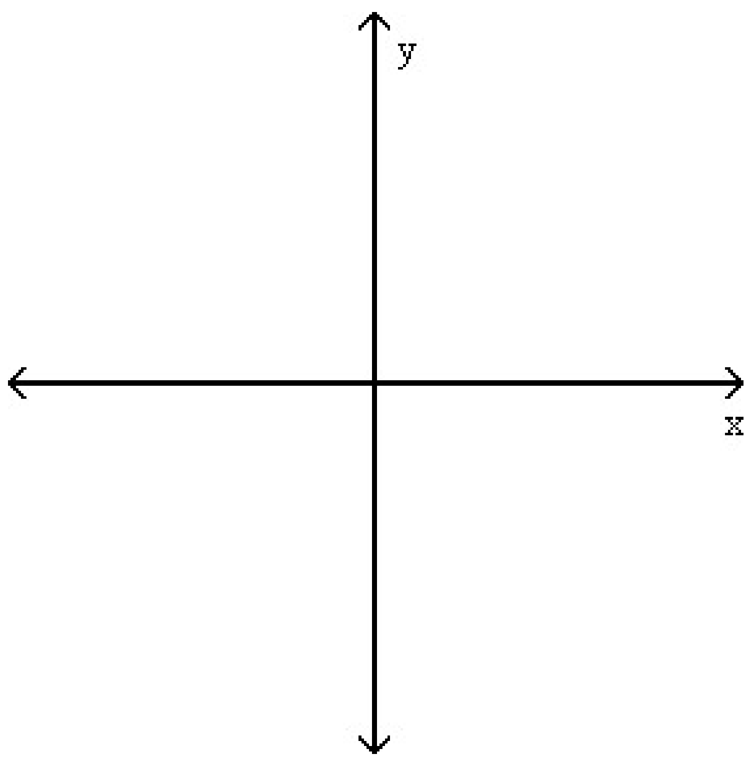
Find where $3x^2 - 10x \geq -7$.



Definition (Horizontal Parabolas)

Example 5

Graph $x = -3y^2 - 25y - 28$.



SECTION 3.2 – POLYNOMIAL FUNCTIONS OF HIGHER DEGREE

Definition (Degree, Coefficients, Leading Term, & Leading Coefficient)

Example 1
Identify the degree, coefficients, leading term, and leading coefficient of each of the following functions.

$f(x) = 2x + 4$

$g(x) = 2x + 4 - 6x^3$

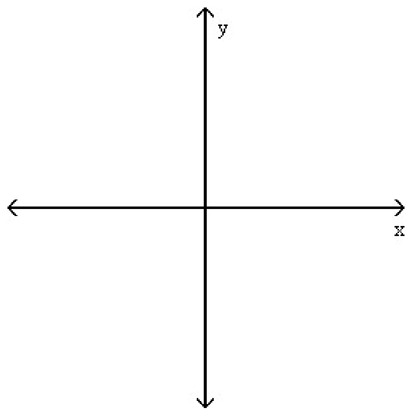
$h(x) = 47x^7 - 2x^4 + 17$

Definition (Power Function)

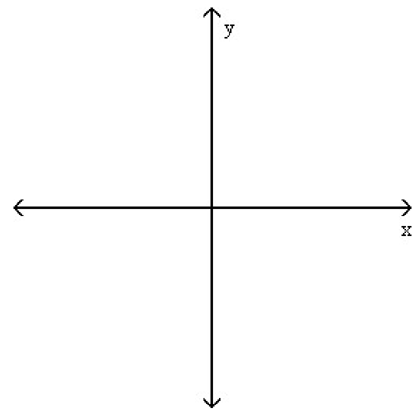
Example 2

Draw the graphs of $f(x) = x^2$, $g(x) = x^3$, $h(x) = x^4$, and $k(x) = x^5$.

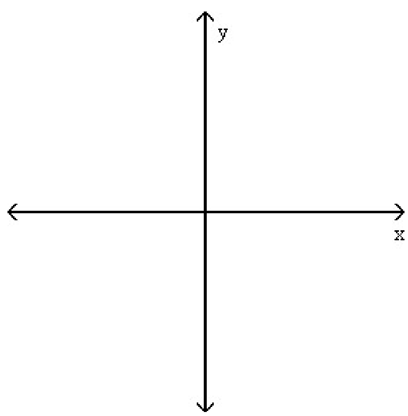
$f(x) = x^2$



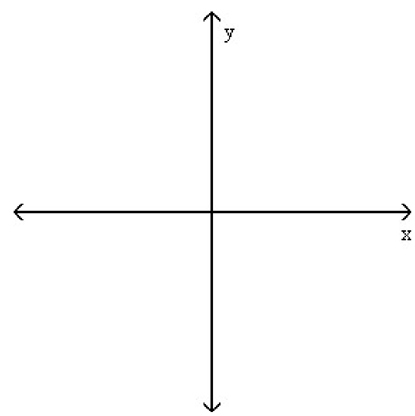
$g(x) = x^3$



$h(x) = x^4$



$k(x) = x^5$



Characteristics of the Graph of $f(x) = x^n$, $n \neq 1$

	n is even	**n is odd**
Symmetry		
Shape of Graph		
Contains These Points		
Monotonicity		

Definition (Continuous)

Theorem

Definition (Turning Points)

Theorem

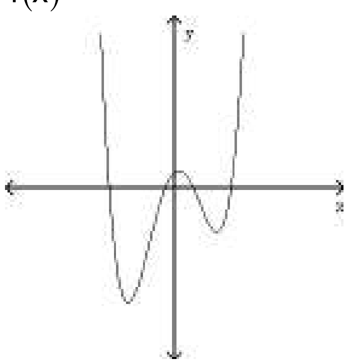
Definition (Limiting Behavior)

Notation

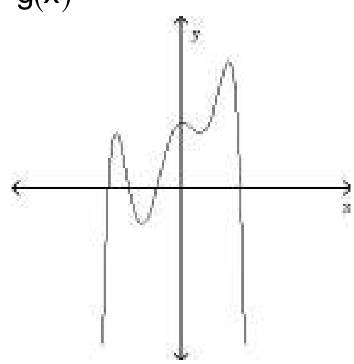
Book's Notation	$f(x) \to \infty$ as $x \to \infty$	$f(x) \to \infty$ as $x \to -\infty$	$f(x) \to -\infty$ as $x \to \infty$	$f(x) \to -\infty$ as $x \to -\infty$
Our Notation	$\lim\limits_{x\to\infty} f(x) = \infty$	$\lim\limits_{x\to-\infty} f(x) = \infty$	$\lim\limits_{x\to\infty} f(x) = -\infty$	$\lim\limits_{x\to-\infty} f(x) = -\infty$

Example 3
Identify the limiting behavior of the following functions.

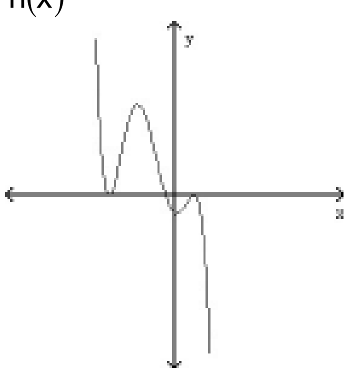
$f(x)$



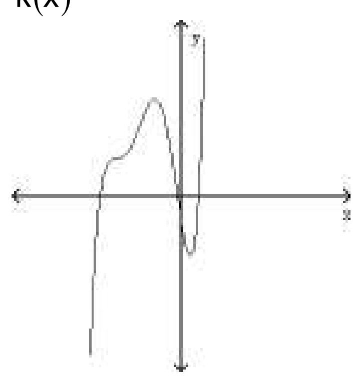
$g(x)$



$h(x)$



$k(x)$

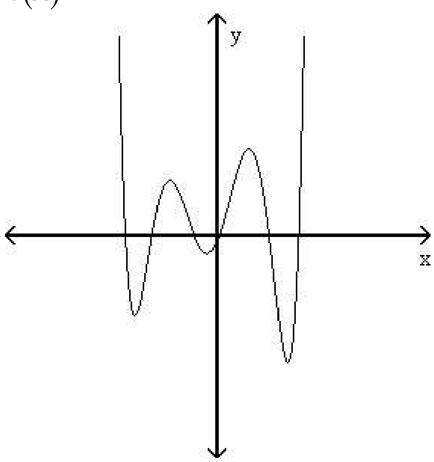


Theorem

Example 4
Identify the limiting behavior of $f(x) = -42x^7 + 7x^6 + x^2 - x + 18$.

Example 5
What is the smallest possible degree of this function? What is the sign of the leading coefficient?

$f(x)$



SECTION 3.3 – POLYNOMIALS AND THEIR DIVISION PROPERTIES

Definition (Divisor, Dividend, Quotient, & Remainder)

Example 1: Long Division

Find the quotient $Q(x)$ and the remainder $R(x)$ if $f(x) = 4x^2 + 20x + 24$ is divided by $D(x) = 2x + 1$.

Example 2: Synthetic Division

Find the quotient $Q(x)$ and the remainder $R(x)$ if $f(x) = 4x^2 + 20x + 24$ is divided by $D(x) = x + 1$.

Find the quotient $Q(x)$ and the remainder $R(x)$ if $g(x) = 2x^3 - 3x - 14x^2 + 21$ is divided by $D(x) = x - 7$.

Remainder Theorem

Factor Theorem

Example 3

Determine if $x - c$ is a factor of $f(x)$.

1.) $c = 2$ & $f(x) = 10x^5 + 3x^2 - 10x^4 - 12$

2.) $c = 1$ & $f(x) = x^4 + 4x^2 + 3$

Definition (Zero of a Function)

Equivalent Statements

Theorem

Rational Zero Theorm

Example 4

Completely factor $f(x)$.

1.) $f(x) = 2x^4 + x^3 - 17x^2 - 4x + 6$

2.) $f(x) = x^5 - 5x^4 + \frac{7x^3}{2} + \frac{13x^2}{2} + \frac{3x}{2} + \frac{9}{2}$

Definition (Multiplicity)

Even-Odd Rule

Odd Balls Don’t Bounce

Example 5

Determine if c is a root of $f(x)$. If it is, determine its multiplicity and what the graph does at c.

1.) $c = 1$ & $f(x) = x^4 - 3x^3 + 9x^2 - 13x + 6$

2.) $c = 3$ & $f(x) = x^4 - 8x^3 + 18x^2 - 27$

Example 6
Find three different examples of functions of degree 8 whose only roots are –5, 2, 3, and 17.

Intermediate Value Property

Example 7
Show that $f(x) = 7x^5 - 3x^4 + 9x^3 + 6x^2 - 7x + 5$ has at least one real root.

The Bisection Method

Example 8
Use the bisection method to approximate the root r of the function f to two decimal places.

$f(x) = 8x^4 - 2x^2 + 5x - 1;\ 0 \le r \le 1$

SECTION 3.4 – RATIONAL FUNCTIONS

Definition (Rational Function)

Example 1
Find the domain of each of the following rational functions.

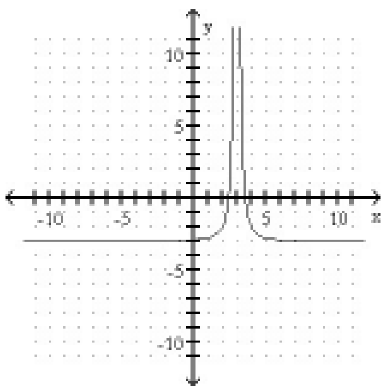
$f(x)=\dfrac{4x^2+3x+2}{x+1}$	$g(x)=\dfrac{16x^5-3x+42}{2x^2+3}$

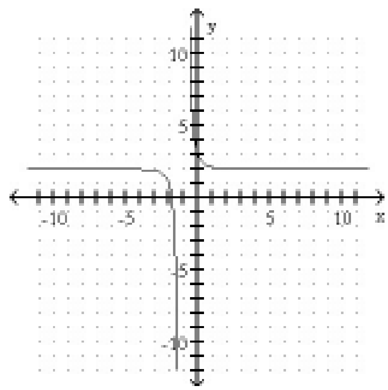
Definition (Vertical Asymptote)

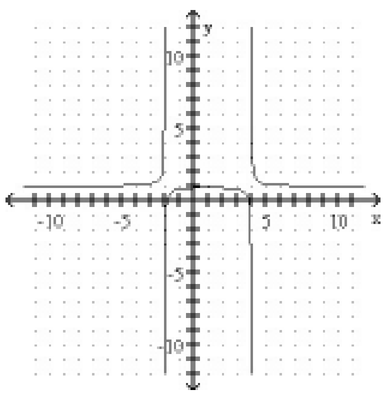
Notation

Book's Notation	$f(x)\to\infty$ as $x\to k$ from the right	$f(x)\to\infty$ as $x\to k$ from the left	$f(x)\to-\infty$ as $x\to k$ from the right	$f(x)\to-\infty$ as $x\to k$ from the left
Our Notation	$\lim\limits_{x\to k^+} f(x)=\infty$	$\lim\limits_{x\to k^-} f(x)=\infty$	$\lim\limits_{x\to k^+} f(x)=-\infty$	$\lim\limits_{x\to k^-} f(x)=-\infty$

Example 2
Identify the vertical asymptotes of each of the following functions.





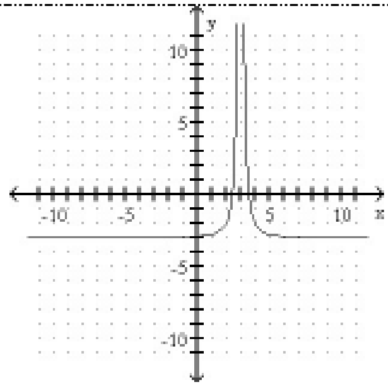


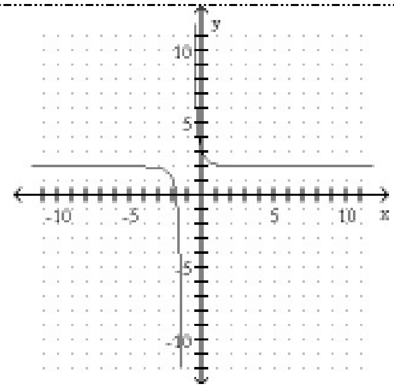
Definition (Horizontal Asymptote)

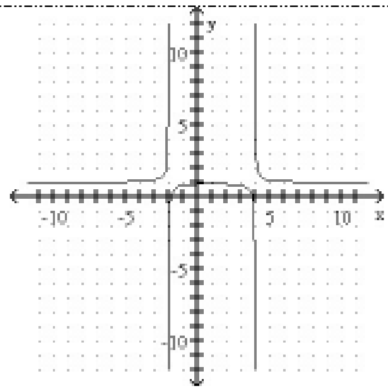
Notation

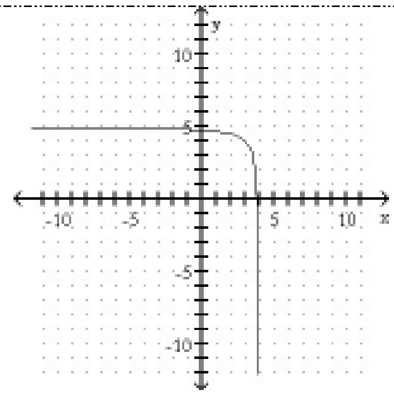
Book's Notation	$f(x) \to k$ as $x \to \infty$	$f(x) \to k$ as $x \to -\infty$
Our Notation	$\lim_{x\to\infty} f(x) = k$	$\lim_{x\to-\infty} f(x) = k$

Example 3
Identify the horizontal asymptotes of each of the following functions.





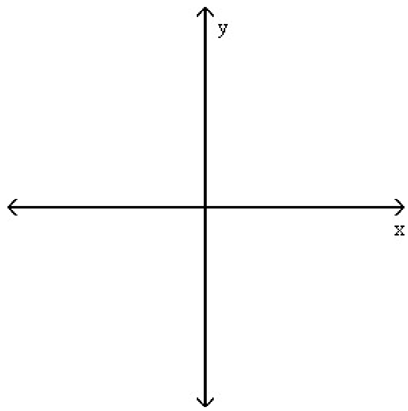




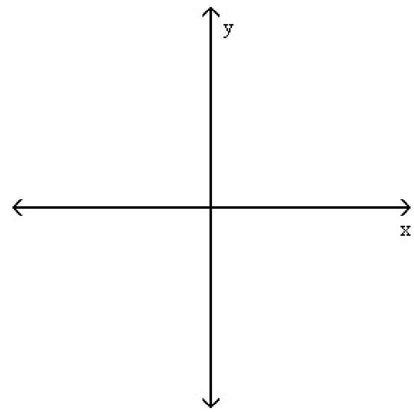
Example 4

Draw the graphs of $f(x)=\frac{1}{x}$, $g(x)=\frac{1}{x^2}$, $h(x)=\frac{1}{x^3}$, and $k(x)=\frac{1}{x^4}$.

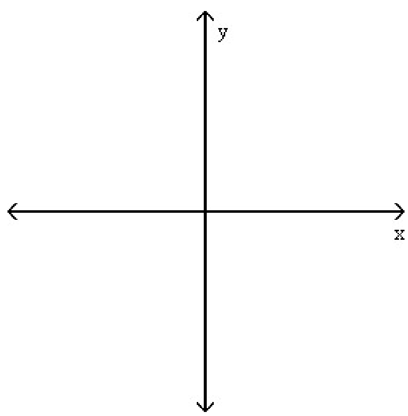
$f(x)=\frac{1}{x}$



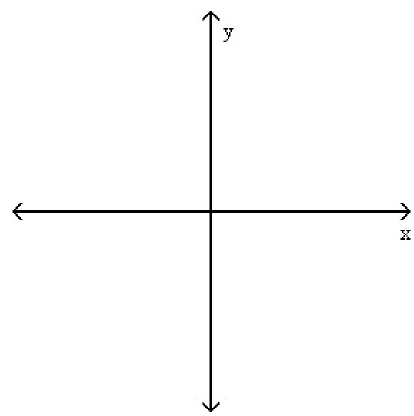
$g(x)=\frac{1}{x^2}$



$h(x)=\frac{1}{x^3}$



$k(x)=\frac{1}{x^4}$

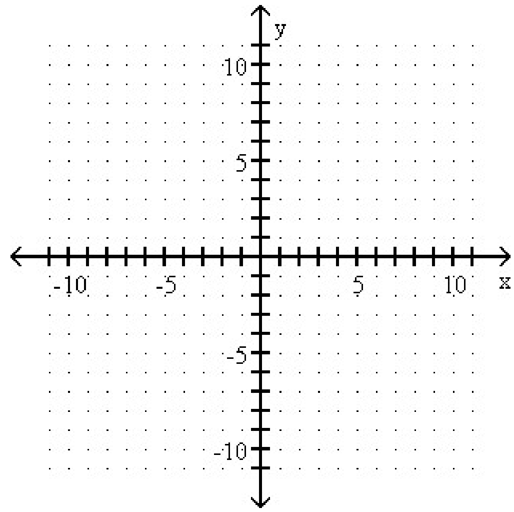


Characteristics of the Graph of $f(x)=\frac{1}{x^n}$ (n is an integer & $n \geq 1$)

	n is even	**n is odd**
Vertical Asymptote		
Horizontal Asymptote		
Symmetry		
Intercepts		

Example 5

Draw the graph of $f(x)=\frac{-2}{(x+1)^5}+3$.



Note:

Property 1

Let $R(x)=\frac{p(x)}{q(x)}$ be a rational function.

- If d is a root of $q(x)$ but not a root of $p(x)$, then R has a vertical asymptote at $x=d$.
- If d is a root of both $p(x)$ and $q(x)$, then R has either a vertical asymptote or a hole at $x=d$. If the multiplicity of d as a root of $p(x)$ greater than or equal to the multiplicity of d as a root of $q(x)$, then R has a hole at $x=d$. If d has a higher multiplicity as a root of $q(x)$ than as a root of $p(x)$, then R has a vertical asymptote at $x=d$.

If you cancel all factors possible without changing the domain of R, then the theorem becomes:

- If d is a root of $q(x)$ but not a root of $p(x)$, then R has a vertical asymptote at $x=d$.
- If d is a root of both $p(x)$ and $q(x)$, then R has a hole at $x=d$.

Example 6

Find all asymptotes, intercepts, and holes of each of the following rational functions.

$$f(x)=\frac{2x^2+7x+3}{x^2-x+2}$$

$$g(x)=\frac{x^3+3x^2+2x}{x^4+4x^3+3x^2}$$

Property 2

Graphing Strategy for Rational Functions

1.) Find all intercepts.

2.) Find all asymptotes & holes.

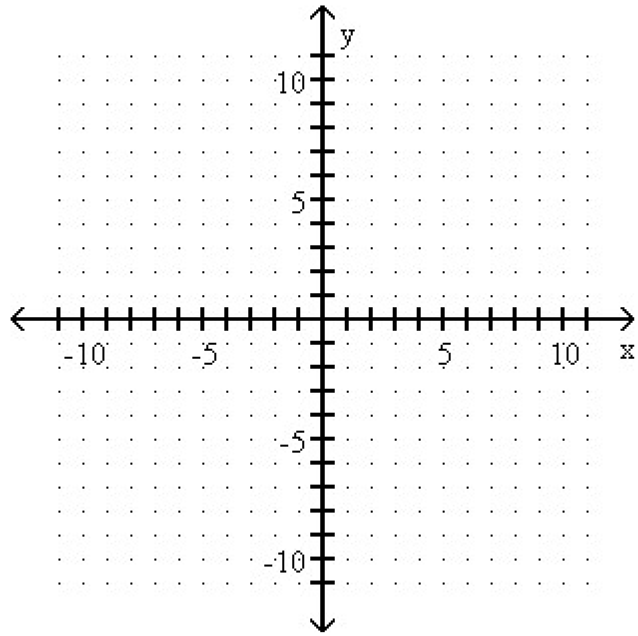
3.) Find any symmetry.

4.) Plot some points.

5.) Draw the graph.

Example 7

Draw the graph of $k(x) = \dfrac{6x-1}{x^2+x-2}$.



CHAPTER 4

SECTION 4.1 – INVERSE FUNCTIONS

Definitions
- Inverse, Invertible Functions
- One-to-One

Concepts/Theorems
- Graphing a Function and Its Inverse
- Identifying Domain and Range of Inverses
- Horizontal Line Test
- Finding f^{-1}

SECTION 4.2 – EXPONENTIAL FUNCTIONS

Definitions
- Exponential Functions
- e
- Natural Exponential Function

Concepts/Theorems
- Graphing Exponential Functions
- Characteristics of Exponential Functions
- Compound Interest Formula
- Continuously Compounded Interest Formula
- Exponential Growth/Decay
- Continuous Growth/Decay

SECTION 4.3 –PROPERTIES OF LOGARITHMIC FUNCTIONS

Definitions
- Logarithm

Concepts/Theorems
- One to One Property
- Properties of Logarithms
- Simplifying Logarithmic Expressions
- Change of Base Formula

SECTION 4.4 – GRAPHING LOGARITHMIC FUNCTIONS

Definitions
- Logarithmic Function

Concepts/Theorems
- Finding Domain of Logarithmic Functions
- Properties of the Graph of $f(x) = \log_b x$

SECTION 4.5 – EXPONENTIAL & LOGARITHMIC EQUATIONS & INEQUALITIES

Concepts/Theorems
- Solving Equations and Inequalities

SECTION 4.1 – INVERSE FUNCTIONS

Definition (Inverse & Invertible Functions)

Example 1

Determine if f & g are inverses of each other.

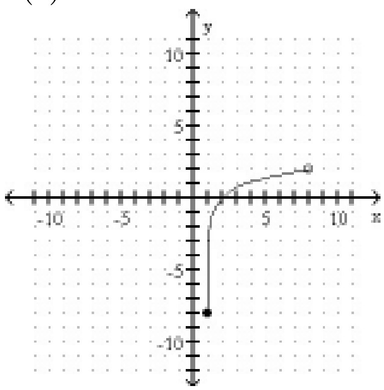
1.) $f(x) = x^3$ & $g(x) = \sqrt[3]{x}$

2.) $f(x) = 2x - 5$ & $g(x) = \dfrac{x}{2} + 5$

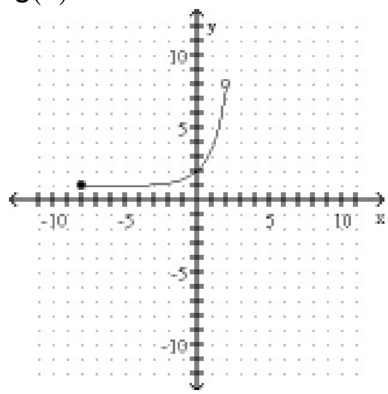
Theorem

Example 2
Determine if f & g are inverses of each other.

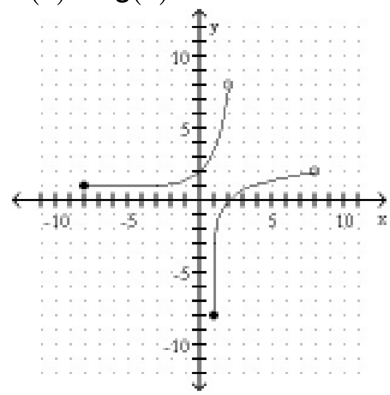
f(x)



g(x)



f(x) & g(x)

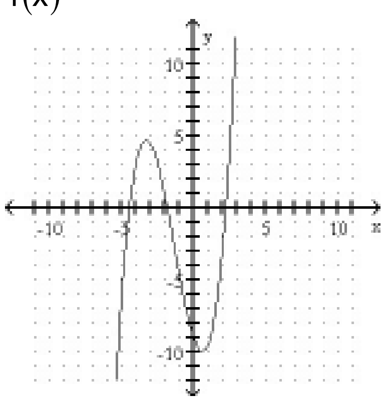


Theorem

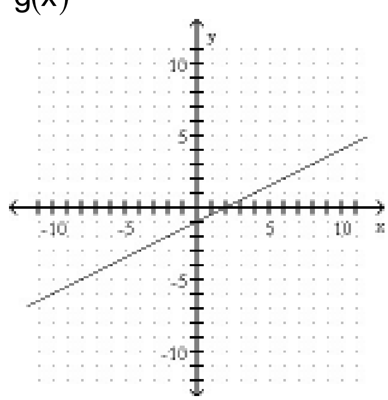
Theorem – Horizontal Line Test

Example 3
Determine if the following graphs represent an invertible function.

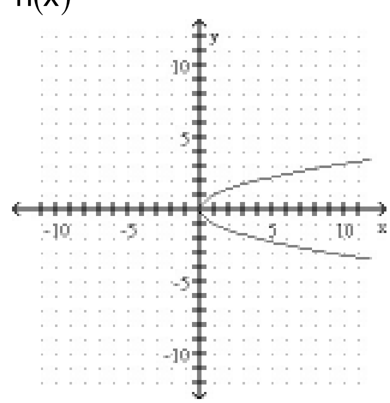
f(x)



g(x)



h(x)



Definition (One-to-One)

Example 4

Determine if each of the following functions is one-to-one.

$f(x) = 5 - x$

$g(x) = 3x^2 + 2x + 1$

Method For Finding $f^{-1}(x)$ (If It Exists)

1.) Replace $f(x)$ with y.

2.) Solve for x.

3.) Switch x & y.

4.) Replace y with $f^{-1}(x)$.

Example 5

Find the inverse of each of the following functions and state the domain and range of each function and its inverse.

$f(x) = 6x + 5$

$g(x) = 7x^3 + 4$

$h(x) = \sqrt{x + 1}$

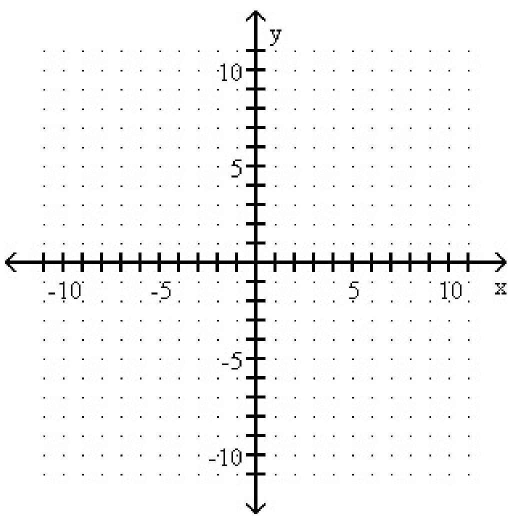
SECTION 4.2 – EXPONENTIAL FUNCTIONS

Definition (Exponential Function)

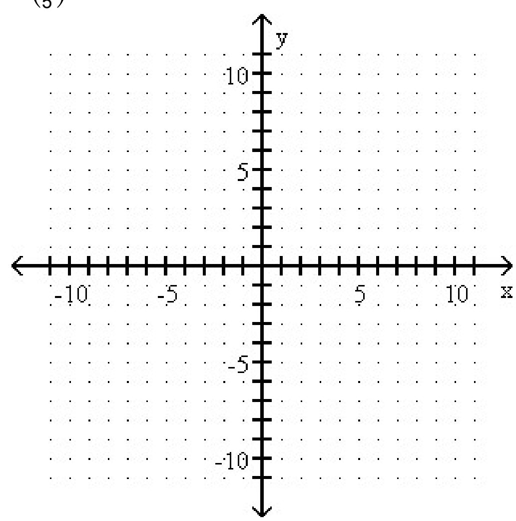
Example 1

Draw the graphs of $f(x) = 3^x$ and $g(x) = \left(\frac{1}{5}\right)^x$.

$f(x) = 3^x$



$g(x) = \left(\frac{1}{5}\right)^x$



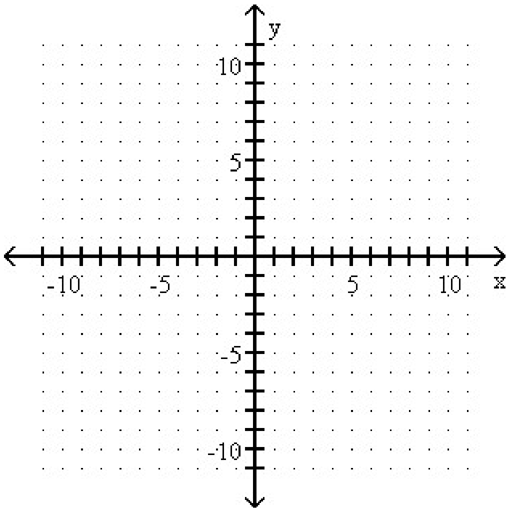
Definition (e)

Definition (Natural Exponential Function)

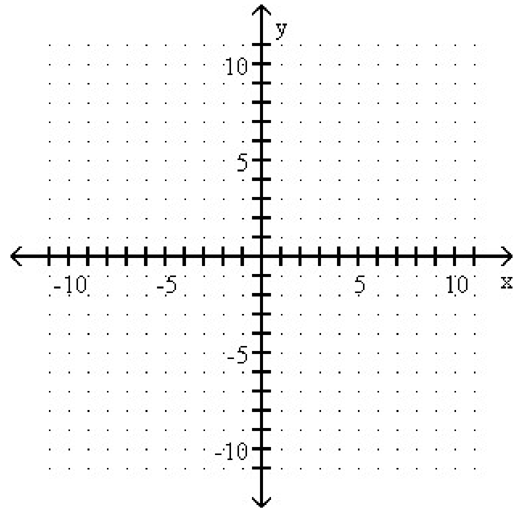
Example 2

Draw the graphs of $f(x) = e^{x}$ and $g(x) = e^{-x+3} - 2$.

$f(x) = e^{x}$



$g(x) = e^{-x+3} - 2$



Characteristics of the Graph of $f(x) = b^{x}$

	$0 < b < 1$	$b > 1$
Domain		
Range		
Monotonicity		
Horizontal Asymptote		
Invertibility		
Intercepts		

Compound Interest Formula

Example 3

Johnny invested $1200 in a bank that pays an annual interest rate of 3.5%.

a.) How much money will Johnny have accumulated after 2 years if interest is compounded yearly?

b.) How much money will Johnny have accumulated after 2 years if interest is compounded quarterly?

c.) How much money will Johnny have accumulated after 2 years if interest is compounded monthly?

d.) How much money will Johnny have accumulated after 10 years if interest is compounded yearly?

e.) How much money will Johnny have accumulated after 10 years if interest is compounded quarterly?

f.) How much money will Johnny have accumulated after 10 years if interest is compounded monthly?

Example 4
Susie accumulated $1500 by leaving her money in a bank for 6 years at an annual interest rate of 6.3% compounding weekly. What was the amount of money she initially put in the bank?

Continuously Compounded Interest Formula

Example 5
Katie invested $1600 at an annual interest rate of 15% compounded continuously. How much money has Katie accumulated after 6 months?

Example 6
Billy accumulated \$2650 by investing his money in an account with an annual interest rate of 6.5% compounded continuously for 3 years and 4 months. How much money did Billy initially invest?

Exponential Growth & Decay

Exponential growth and decay can be described by a function of the form $f(t) = Pa^t$ where $f(t)$ is the quantity at time t, P is the initial quantity (the amount when $t = 0$), and a is the factor by which the quantity changes.

- If the quantity is growing at a rate r per time period, then $a > 1$ and $a = 1 + r$.
- If the quantity is decaying at a rate r per time period, then $0 < a < 1$ and $a = 1 - r$.

Example 7
The fruit fly population in a certain laboratory triples every day. Today there are 200 fruit flies.
a.) Make a table showing the number of fruit flies present on the first 4 days.

b.) Find a function of the form $f(x) = Pa^x$ that describes the fruit fly population at time x days.

c.) What is the daily growth factor?

d.) How many fruit flies will there be a week from now?

Continuous Growth/Decay

Example 8
Population in Smallville grows continuously at a rate of .5% per year. If the current population is 45,562, what will it be in 9 years?

Example 9
The Thanksgiving turkey's temperature drops continuously once it is removed from the oven at a rate of 7.4% every 10 minutes. If the temperature of the turkey is 225° an hour after being removed from the oven, what was the temperature of the turkey when it initially came out of the oven?

SECTION 4.3 – PROPERTIES OF LOGARITHMIC FUNCTIONS

Definition (Logarithm)

One-To-One Property

Example 1

$\log_2 \frac{1}{2} =$

$\log_3 27 =$

$\log_b b =$

$\log_b 1 =$

Notation

Example 2
Find the inverse of each of the following functions.
$f(x) = \ln x$

$g(x) = \log_2 x$

Exponent Laws

$x^0 = 1$ (assuming $x \neq 0$)	$x^{-a} = \frac{1}{x^a}$	$x^{a+b} = x^a x^b$
$x^1 = x$	$x^{a\,b} = \left(x^a\right)^b$	$x^{a-b} = \frac{x^a}{x^b}$

Logarithm Properties

Example 3

$\log_b b^r =$

$10^{\log x} =$

$e^{\ln x} =$

$\log_3 \frac{6}{35} + \log_3 \frac{14}{9} - \log_3 \frac{12}{5} =$

Change of Base Formula

Example 4

$\log_2 10 =$

$\log_5 7 =$

$\log_{12} 56 =$

SECTION 4.4 – LOGARITHMIC FUNCTIONS

Definition (Logarithmic Function)

Finding Domain of Logarithmic Functions

If $f(x) = \log_b x$, then the domain of f is $(0, \ \infty)$. That is, we can't have $x \leq 0$.

If $f(x) = \log_b(\text{"stuff"})$, we can't have $x \leq 0$.

Example 1

Find the domain of each of the following functions.

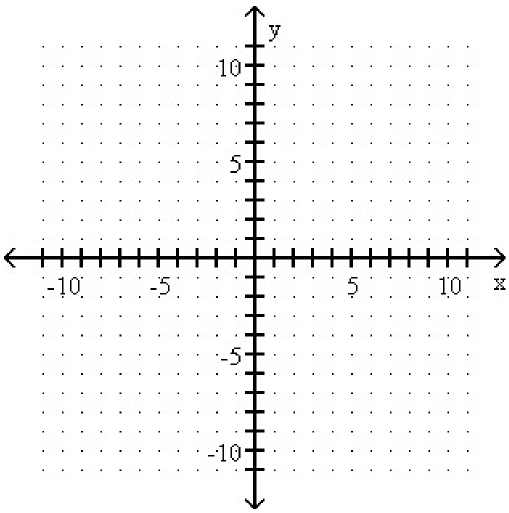
$f(x) = \log_6(4 - 3x)$

$g(x) = \log_2(2x^2 - 9x + 4)$

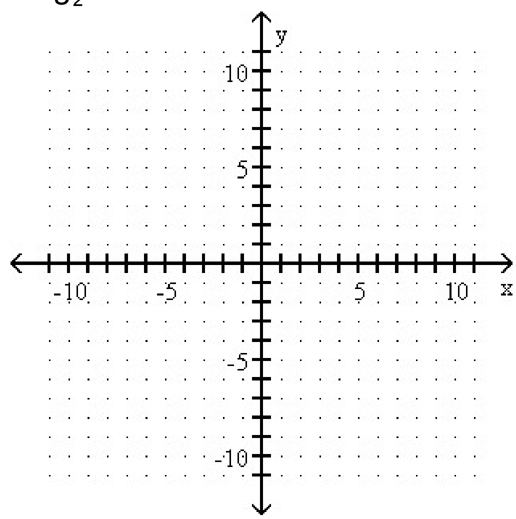
Example 2

Draw the graphs of $f(x) = \log_{1/2} x$ and $g(x) = \log_2 x$.

$f(x) = \log_{1/2} x$



$g(x) = \log_2 x$



Characteristics of the Graph of $f(x) = \log_b x$		
	$0 < b < 1$	$b > 1$
Domain		
Range		
Monotonicity		
Vertical Asymptote		
Invertibility		
Intercepts		
Other		

SECTION 4.5 – EXPONENTIAL & LOGARITHMIC EQUATIONS & INEQUALITIES

Strategy for Solving Equations Involving Logarithms

1.) Rewrite the equation with all of the logs on the left side and everything else on the right side of the equal sign.
2.) Use properties of log's to simplify the left side into one logarithm.
3.) Convert into exponential form.
4.) Solve.

Example 1

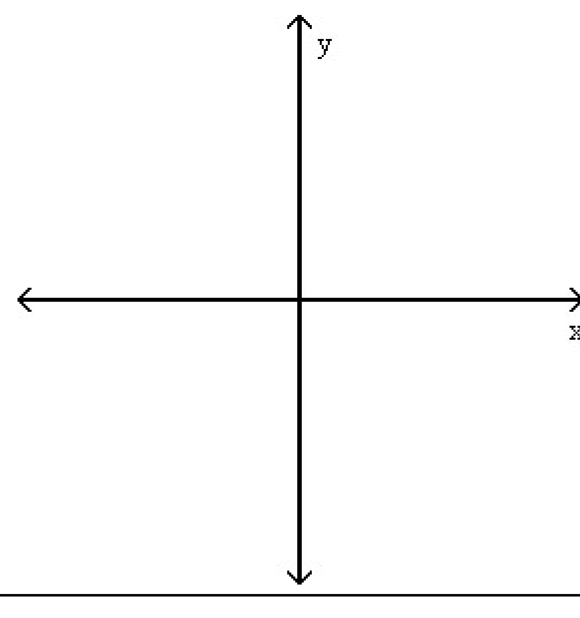
$\log(x+9)-\log x=1$

$\log\sqrt[3]{x^2+21x}=\frac{2}{3}$

$\frac{\ln(x+1)}{\ln(x-1)}=2$

Example 2
Solve $6^{2x-3} = 5^{x+1}$ for x.

Example 3
Solve the following inequality.
$1.25^x < .62^x$



CHAPTER 5

SECTION 5.1 –SYSTEMS OF LINEAR EQUATIONS, A FIRST LOOK

Definitions
- Linear Equation
- Linear System
- Solution
- Solution Set
- Equivalent Systems
- Consistent/Inconsistent System

Concepts/Theorems
- Substitution Method
- Elimination Method

SECTION 5.2 – LARGER SYSTEMS: MATRIX REPRESENTATION & GAUSS-JORDAN ELIMINATION

Definitions
- Matrix
- Element
- Matrix Size
- Coefficient Matrix
- Augmented Matrix
- Echelon Form
- Row Reduced Echelon Form
- Row Reduction
- Equivalent Matrices

Concepts/Theorems
- Row Reduce a Matrix
- Use Matrices to Solve Linear Systems

SECTION 5.3 – MORE MODELING AND APPLICATIONS

Concepts/Theorems
- Solve Applied Problems

SECTION 5.1 –SYSTEMS OF LINEAR EQUATIONS, A FIRST LOOK

Definition (Linear Equation, Linear System, Solution, Solution Set, Equivalent Systems)

Substitution Method

Example 1

Find the solution to the following systems.

1.) $x - 2y = 4$
$3x + y = 5$

2.) $6c - 2d = 4$
$7c + d = 13$

$$\begin{aligned} 2r - 5s + t &= 0 \\ 6r + 4s - 3t &= -12 \\ 2r + 4t &= 6 \end{aligned}$$

3.)

Elementary Equation Operations

1.) ***Replacement:*** Replace one equation by the sum of itself and a multiple of another equation.

2.) ***Interchange:*** Interchange two equations.

3.) ***Scaling:*** Multiply both sides of an equation by the same nonzero constant.

Elimination Method

Example 2

Find the solution to the following systems.

1.) $$\begin{aligned} 9x - 2y &= 5 \\ 3x - 3y &= 11 \end{aligned}$$

2.) $$\begin{aligned} -3m + n &= 2 \\ 7m - 8n &= 1 \end{aligned}$$

3.) $$\begin{aligned} 2a + 4b - c &= -7 \\ -5a + 2b + c &= -5 \\ -a + 4b &= -6 \end{aligned}$$

Example 3

Find the solution to the following systems.

1.) $$\begin{aligned} 2s + 4t &= 8 \\ 3s + 6t &= 3 \end{aligned}$$

2.) $$\begin{aligned} -3x + 8y &= 29 \\ -6x + 16y &= 58 \end{aligned}$$

Example 4
A collection of 42 coins consists of dimes and nickels. The total value is $3. How many dimes and how many nickels are there?

Existence & Uniqueness

SECTION 5.2 – LARGER SYSTEMS: MATRIX REPRESENTATION & GAUSS-JORDAN ELIMINATION

Definition (Matrix, Element, Matrix Size)

Definition (Coefficient Matrix, Augmented Matrix)

Example 1
Find the coefficient and augmented matrices of each of the following systems. Also, identify the size of each matrix.

System	Coefficient Matrix	Augmented Matrix
$\begin{aligned} x-2y&=4 \\ 3x+y&=5 \end{aligned}$		
$\begin{aligned} x-2y+3z&=0 \\ 2x-y-z&=-1 \\ x+y-z&=1 \end{aligned}$		

Definition (Leading Entry)

Echelon Form	**Row Reduced Echelon Form**
1.) All nonzero rows are above any rows of all zeros. 2.) Each leading entry of a row is in a column to the right of the leading entry of the row above it. 3.) All entries in a column below a leading entry are zeros.	1.) All nonzero rows are above any rows of all zeros. 2.) Each leading entry of a row is in a column to the right of the leading entry of the row above it. 3.) All entries in a column below a leading entry are zeros. 4.) The leading entry in each nonzero row is 1. 5.) Each leading 1 is the only nonzero entry in its column.

Example 2
Determine if each of these is an echelon matrix, a row reduced echelon matrix, or neither.

$\begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$	$\begin{pmatrix} 1 & 2 & 2 & 0 \\ 0 & 5 & 4 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$
$\begin{pmatrix} 92 & 65 \\ 0 & 12 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 6 & 5 & 2 \\ 0 & 8 & 1 \\ 0 & 0 & 5 \\ 0 & 0 & 2 \end{pmatrix}$

Elementary Row Operations

1.) ***Replacement:*** Replace one row by the sum of itself and a multiple of another row.

2.) ***Interchange:*** Interchange two rows.

3.) ***Scaling:*** Multiply all entries of a row by a nonzero constant.

Note

Replacement and scaling operations may be combined into more efficient row operations of the form $aR_n + bR_m \to R_n$ where $m \neq n$ and $a \neq 0$.

Definition (Row Reduction)

The Row Reduction Algorithm

1.) Begin with the leftmost nonzero column. Interchange rows, if necessary, so that the leading entry for the top row is in this column.

2.) Use row replacement operations to create zeros in all positions below this leading entry.

3.) Cover the row and column containing this leading entry (and any rows above it or columns to the left of it) and apply steps 1-3 to the submatrix that remains. Repeat this process until there are no more nonzero rows to modify.

Your matrix should now be in echelon form.

4.) Beginning with the rightmost leading entry and working upward and to the left, create zeros above each leading entry. If the leading entry is not 1, use a scaling operation to make it a 1.

Your matrix should now be in row reduced echelon form.

Example 3

Row reduce each of these matrices into row reduced echelon form.

$$\begin{pmatrix} 1 & 4 & 0 & 7 \\ 2 & 7 & 0 & 10 \end{pmatrix}$$

$$\begin{pmatrix} 3 & -4 & -5 & 8 \\ 1 & -2 & 1 & 0 \\ -4 & 5 & 9 & -9 \end{pmatrix}$$

Definition (Equivalent Matrices)

Using Row Reduction To Solve A Linear System

1.) Write the augmented matrix for the system.

2.) Use the row reduction algorithm to obtain an equivalent augmented matrix in echelon form. Decide whether the system is consistent. If there is no solution, stop; otherwise continue.

3.) Continue row reduction to obtain the reduced echelon form. Write the system of equations corresponding to the row reduced echelon matrix.

Note
If a leading entry of any row is in the right most column of any matrix that is equivalent to the augmented matrix for a system, then the system is inconsistent. It is most convenient to look only at the leading entries of the echelon form of the augmented matrix to determine if a system is consistent.

Example 4

Find solutions to the following systems using matrices.

$$3x + 4y = 12$$
$$3x - 8y = 0$$

$$6a + 3b = 9$$
$$8a + 4b = -2$$

$$3r + 2s + 2t = 29$$
$$9r + 8s + 9t = 116$$
$$r + 2s + 9t = 86$$

SECTION 5.3 – MORE MODELING AND APPLICATIONS

Example 1
Grace sells two kinds of granola. One is worth $4.05 per pound and the other is worth $2.70 per pound. She wants to blend the two granolas to get a 15 lb mixture worth $3.15 per pound. How much of each kind of granola should be used?

Example 2

Animals in an experiment are to be kept under a strict diet. Each animal is to receive, among other things, 20 grams of protein and 6 grams of fat. The laboratory technician is able to purchase two food mixes. Mix A contains 10% protein and 6% fat and Mix B contains 20% protein and 2% fat. How many grams of each mix should be used to obtain the right diet for a single animal?

Example 3
A 480 m wire is cut into three pieces. The second piece is three times as long as the first. The third is four times as long as the second. How long is each piece?

Example 4

A furniture manufacturer makes chairs, coffee tables, and dining room tables. Each chair requires 10 minutes of sanding, 6 minutes of staining, and 12 minutes of varnishing. Each coffee table requires 12 minutes of sanding, 8 minutes of staining, and 12 minutes of varnishing. Each dining room table requires 15 minutes of sanding, 12 minutes of staining, and 18 minutes of varnishing. The sanding bench is available for 16 hours per week, the staining bench is open for 11 hours per week, and the varnishing bench is available for 18 hours per week. How many (per week) of each type of furniture should be made so that the benches are fully utilized?

Example 5

The sum of the digits in a four-digit number is 10. Twice the sum of the thousands digit and the tens digit is 1 less than the sum of the other two digits. The tens digit is twice the thousands digit. The ones digit equals the sum of the thousands digit and the hundreds digit. Find this four-digit number.

Example 6

A chemist has two saline solutions: one has 10% concentration of saline and the other has 35% concentration of saline. How many cubic centimeters of each solution should be mixed together in order to obtain 60 cubic centimeters of solution with a 25% concentration of saline?

Example 7

A horticulturist wishes to mix three types of fertilizer. Type A contains 25% nitrogen, Type B contains 35% nitrogen, and Type C contains 40% nitrogen. She wants a mixture of 4000 pounds with a final concentration of $35\frac{5}{8}\%$ nitrogen. The final mixture should also contain three times as much of Type C than of Type A. How much of each type is in the final mixture?