SECTION 1.1 – LINEARITY

Example 1
At the beginning of the year, the price of gas was $3.19 per gallon. At the end of the year, the price of gas was $1.52 per gallon. What is the total change in the price of gas?

Example 2
John collects marbles. After one year, he had 60 marbles and after 5 years, he had 140 marbles. What is the average rate of change of marbles with respect to time?

Example 3
On an average summer day in a large city, the pollution index at 8:00 A.M. is 20 parts per million, and it increases linearly by 15 parts per million each hour until 3:00 P.M. Let P be the amount of pollutants in the air x hours after 8:00 A.M.

a.) Write the linear model that expresses P in terms of x.

b.) What is the air pollution index at 1:00 P.M.?

c.) Graph the equation P for $0 \le x \le 7$.

SECTION 1.2 – GEOMETRIC & ALGEBRAIC PROPERTIES OF A LINE

Example 1

Graph the line containing the point P with slope m.

a.) $P = (1,\ 1)$ & $m = -3$

b.) $P = (2,\ 1)$ & $m = \frac{1}{2}$

c.) $P = (-1,\ -3)$ & $m = \frac{3}{5}$

d.) $P = (0,\ -2)$ & $m = -\frac{2}{3}$

Example 2

Find the equation of the line through the points P and Q.

a.) $P = (1,\ 1)$ & $Q = (9,\ 12)$

b.) $P = (-2,\ -5)$ & $Q = (3,\ 2)$

c.) $P = (-1,\ 2)$ & $Q = (6,\ -4)$

d.) $P = (0,\ 6)$ & $Q = (-3,\ -2)$

Example 3

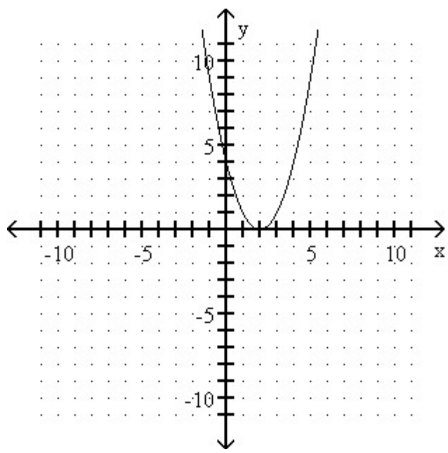
a.) Find the equation of a line that is parallel to the line $y = -\frac{6}{7}x - 19$.

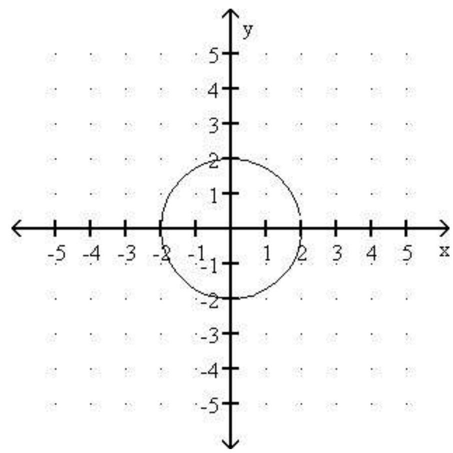
b.) Find the equation of a line that is perpendicular to the line $y = -\frac{6}{7}x - 19$.

c.) Find the equation of the line that is parallel to the line $y = -\frac{6}{7}x - 19$ and goes through the point $(2,\ -1)$.

d.) Find the equation of the line that is perpendicular to the line $y = -\frac{6}{7}x - 19$ and goes through the point $(2,\ -1)$.

Example 4
Find the intercepts of each of the following graphs.





Example 5
Find all intercepts of the following equations.

a.) $x^2 + y^2 = 10$

b.) $y = 2x^2 + 6$

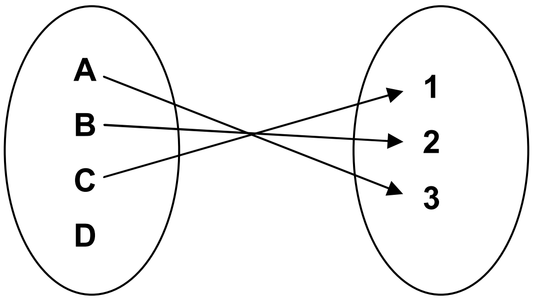
c.) $y = x^3$

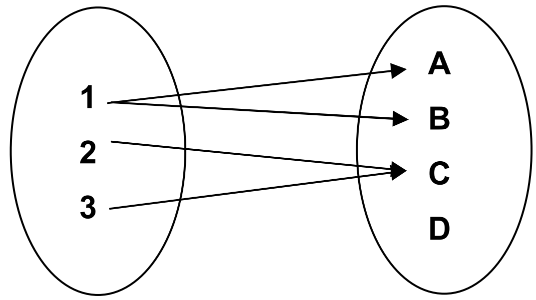
d.) $y = 3 - 3x^3$

SECTION 1.3 – FUNCTIONS AND MODELING

Example 1
Voting for President can be thought of as a function. Assuming that everyone who can vote does vote, then the domain is the set of American citizens who are age 18 or older and the range is the set of all Presidential nominees who receive at least one vote. Notice that each voter can only vote for one candidate. That is, for each input there is exactly one output.

Example 2
Determine if each of these is a function. Also, identify the inputs, outputs, domain, and range where applicable.





Example 3
The following table displays the number of students registered in each section of underwater basket weaving.

Section # (S)	1	2	3	4	5	6	7	8	9	10
# Of Students Enrolled (E)	25	32	19	22	41	16	32	22	27	31

Is E a function of S?	
What are the inputs?	
What are the outputs?	
What is the domain?	
What is the range?	
Is S a function of E?	

Example 4 - Evaluating a Function
Let $f(x) = 4x^2 + 3x + 2$. Find each of the following.

$f(0) =$

$f(2) =$

$f(\frac{1}{2}) =$

$f(2k + 5) =$

$f(\_) =$

$f(\Delta) =$

$\dfrac{f(t+h) - f(t)}{h} =$

Example 5 - Finding Domain
Find the domain of each of the following functions.

$f(x) = 2x + 4$

$g(x) = \sqrt[6]{2x - 3}$

$h(x) = \dfrac{4x + 3}{x^2 - 1}$

$k(x) = \dfrac{\sqrt{1 + 6x}}{2x^2 - 3x - 2}$

Example 6
Determine if each of the following equations describes y as a function of x, x as a function of y, or both.

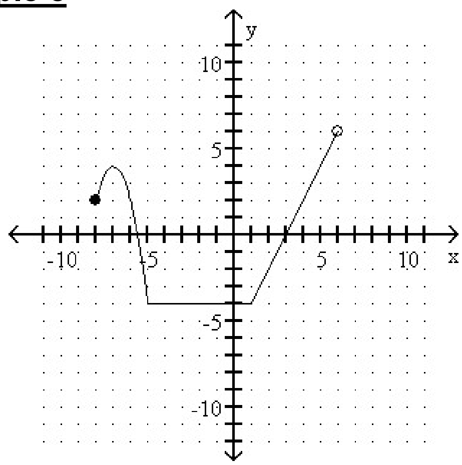
$4x + 3y = 2$

$4x^2 - 3y = 6$

$2x + 4 = y^2$

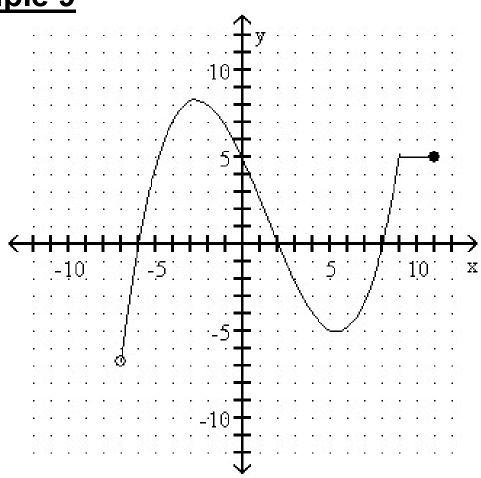
Example 7
A ball is thrown straight up into the air. The height of the ball in feet above the ground is described by the function $h(t) = -2t^2 + 14$ where time is measured in seconds. Find the total change and average rate of change in height after 2 seconds and interpret each.

Example 8



Domain:	
Range:	
y-intercept:	
x-intercepts:	

Example 9



Domain:	
Range:	
y-intercept:	
x-intercepts:	

Example 10

A manufacturing company wants to make digital cameras and wholesale the cameras to retail outlets throughout the U.S. The company's financial department has come up with the following price-demand and cost data where p is the wholesale price per camera at which x million cameras are sold and C is the cost in millions of dollars for manufacturing and selling x million cameras.

x (millions)	p ($)
2	84.80
5	69.80
8	54.80
10	44.80
13	29.80

x (millions)	C (thousands of $)
1	175.7
3	215.1
6	274.2
11	372.7
14	431.8

a.) Assuming that x and p are linearly related, write a formula that expresses p as a function of x. You may also assume that $0 \le x \le 15$.

b.) Assuming that x and C are linearly related, write a formula that expresses C as a function of x. You may also assume that $0 \le x \le 15$.

c.) What are the revenue and profit functions?

d.) At what point(s) does the company break-even, make a profit, or experience a loss?

e.) When does the company experience its maximum profits?

SECTION 2.1 – SIMPLE INTEREST AND COMPOUND INTEREST

Example 1
Find the total amount due on a loan of $800 at 9% simple interest at the end of 4 months.

Example 2
If you want to earn an annual rate of 10% on your investments, how much (to the nearest cent) should you pay for a note that will be worth $5000 in 9 months?

Example 3
Treasury bills are one of the instruments the U.S. Treasury Dept uses to finance the public debt. If you buy a 180-day treasury bill with a maturity value of $10,000 for $9,893.78, what annual simple interest rate will you earn?

Example 4
You want to invest $1000 at 8% interest for 5 years.

a.) How much money will you have if interest is compounded semiannually?
b.) How much money will you have if interest is compounded monthly?
c.) How much money will you have if interest is compounded daily?

Example 5
How much should you invest now at 10% interest compounded quarterly to have $8000 toward the purchase of a car in 5 years?

Example 6
If money placed in a certain account triples in 2 years when interest is compounded quarterly, then what is the annual interest rate?

Example 7
A $10,000 investment in a particular growth-oriented mutual fund over a recent 10 year period would have grown to $126,000. What annual nominal rate compounded monthly would produce the same growth? What is the annual percentage yield?

Example 8
Find the APY's for each of the banks in the following table and compare the CDs.

CERTIFICATES OF DEPOSIT		
BANK	RATE	COMPOUNDED
Advanta	4.95%	Monthly
DeepGreen	4.96%	Daily
Charter One	4.97%	quarterly

Example 9
A savings and loan wants to offer a CD with a monthly compounding rate that has an effective annual rate of 7.5%. What annual nominal rate compounded monthly should they use?

SECTION 2.2 – FUTURE VALUE OF AN ANNUITY

Example 1
What is the value of an annuity at the end of 20 years if $2000 is deposited each year into an account earning 8.5% interest compounded annually? How much of the value is interest?

Example 2
A person makes monthly deposits of $100 into an ordinary annuity. After 30 years, the annuity is worth $160,000. What annual rate compounded monthly has this annuity earned during this 30-year period?

Example 3
A company estimates that it will have to replace a piece of equipment at a cost of $800,000 in 5 years. To have this money available in 5 years, a sinking fund is established by making equal monthly payments into an account paying 6.6% compounded monthly. How much should each payment be? How much interest in earned during the last year?

Example 4
Jane deposits $2000 annually into a Roth IRA that earns 6.85% compounded annually. (The interest earned by a Roth IRA is tax free.) Due to a change in employment, these deposits stop after 10 years, but the account continues to earn interest until Jane retires 25 years after the last deposit was made. How much is in the account when Jane retires?

SECTION 2.3 – PRESENT VALUE OF AN ANNUITY

Example 1
What is the present value of an annuity that pays $200 per month for 5 years if money is worth 6% compounded monthly?

Example 2
Recently, Lincoln Benefit Life offered an ordinary annuity that earned 6.5% compounded annually. A person plans to make equal annual deposits into this account for 25 years in order to then make 20 equal annual withdrawals of $25,000, reducing the balance in the account to zero. How much must be deposited annually to accumulate sufficient funds to provide for these payments? How much total interest is earned during this entire 45-year process?

SECTION 2.4 – BORROWING

Example 1
Suzie borrows $10,000 from her parents to buy a car. She agrees to pay them in equal monthly payments over the next 5 years with a simple interest rate of 5%. How much will Suzie pay each month?

Example 2
A loan is discounted over 30 months at an annual simple interest rate of 10%.
- a.) Find the proceeds if the amount of the loan is $1500.
- b.) Find the amount of the loan if the proceeds are $1500.

Example 3
Assume that you buy a television set for $800 and agree to pay for it in 18 equal monthly payments at 1.5% interest per month on the unpaid balance. How much are your payments? How much interest will you pay?

Example 4
You have negotiated a price of $25,200 for a new Bison pickup truck. Now you must choose between 0% financing for 48 months or a $3000 rebate. If you choose the rebate, you can obtain a loan for the balance at 4.5% interest compounded monthly for 48 months at your credit union. Which option should you choose?

Example 5
If you borrow $500 that you agree to repay in six equal monthly payments at 1% interest per month on the unpaid balance, how much of each monthly payment is used for interest and how much is used to reduce the unpaid balance?

Example 6
A couple plans to buy a home for $200,000. They have put $50,000 down and will obtain a mortgage for $150,000 at an interest rate of 10.5% compounded monthly. They must decide whether to apply for a 30-year or a 15-year mortgage. Find the monthly payment and total interest paid for both the 30-year mortgage and the 15-year mortgage.

SECTION 3.1 –SYSTEMS OF LINEAR EQUATIONS, A FIRST LOOK

Example 1 – Substitution Method
Find the solution to the following systems.

1.) $x - 2y = 4$
$3x + y = 5$

2.) $6c - 2d = 4$
$7c + d = 13$

3.) $2a - 5b + c = 0$
$6a + 4b - 3c = -12$
$2a + 4c = 6$

Example 2 – Elimination Method
Find the solution to the following systems.

1.) $9x - 2y = 5$
$3x - 3y = 11$

2.) $-3m + n = 2$
$7m - 8n = 1$

3.) $2r + 4s - t = -7$
$-5r + 2s + t = -5$
$-r + 4s = -6$

Example 3
Find the solution to the following systems.

1.) $2s + 4t = 8$
$3s + 6t = 3$

2.) $-3x + 8y = 29$
$-6x + 16y = 58$

Example 4
A collection of 42 coins consists of dimes and nickels. The total value is $3. How many dimes and how many nickels are there?

SECTION 3.2 – LARGER SYSTEMS: MATRIX REPRESENTATION & GAUSS-JORDAN ELIMINATION

Example 1
Find the coefficient and augmented matrices of each of the following systems. Also, identify the size of each matrix.

System	Coefficient Matrix	Augmented Matrix
$x - 2y = 4$ $3x + y = 5$		
$x - 2y + 3z = 0$ $2x - y - z = -1$ $x + y - z = 1$		

Example 2
Determine if each of these is an echelon matrix, a row reduced echelon matrix, or neither.

$\begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$	$\begin{pmatrix} 1 & 2 & 2 & 0 \\ 0 & 5 & 4 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$
$\begin{pmatrix} 92 & 65 \\ 0 & 12 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 6 & 5 & 2 \\ 0 & 8 & 1 \\ 0 & 0 & 5 \\ 0 & 0 & 2 \end{pmatrix}$

Example 3
Row reduce each of these matrices into row reduced echelon form.

1.) $\begin{pmatrix} 1 & 4 & 0 & 7 \\ 2 & 7 & 0 & 10 \end{pmatrix}$

2.) $\begin{pmatrix} 3 & -4 & -5 & 8 \\ 1 & -2 & 1 & 0 \\ -4 & 5 & 9 & -9 \end{pmatrix}$

Example 4

Find solutions to the following systems using matrices.

1.) $3x + 4y = 12$

$3x - 8y = 0$

2.) $6a + 3b = 9$

$8a + 4b = -2$

3.) $3r + 2s + 2t = 29$

$9r + 8s + 9t = 116$

$r + 2s + 9t = 86$

SECTION 3.3 – MORE MODELING AND APPLICATIONS

Example 1
Grace sells two kinds of granola. One is worth $4.05 per pound and the other is worth $2.70 per pound. She wants to blend the two granolas to get a 15 lb mixture worth $3.15 per pound. How much of each kind of granola should be used?

Example 2
Animals in an experiment are to be kept under a strict diet. Each animal is to receive, among other things, 20 grams of protein and 6 grams of fat. The laboratory technician is able to purchase two food mixes. Mix A contains 10% protein and 6% fat and Mix B contains 20% protein and 2% fat. How many grams of each mix should be used to obtain the right diet for a single animal?

Example 3
A 480 m wire is cut into three pieces. The second piece is three times as long as the first. The third is four times as long as the second. How long is each piece?

Example 4
A furniture manufacturer makes chairs, coffee tables, and dining room tables. Each chair requires 10 minutes of sanding, 6 minutes of staining, and 12 minutes of varnishing. Each coffee table requires 12 minutes of sanding, 8 minutes of staining, and 12 minutes of varnishing. Each dining room table requires 15 minutes of sanding, 12 minutes of staining, and 18 minutes of varnishing. The sanding bench is available for 16 hours per week, the staining bench is open for 11 hours per week, and the varnishing bench is available for 18 hours per week. How many (per week) of each type of furniture should be made so that the benches are fully utilized?

Example 5
The sum of the digits in a four-digit number is 10. Twice the sum of the thousands digit and the tens digit is 1 less than the sum of the other two digits. The tens digit is twice the thousands digit. The ones digit equals the sum of the thousands digit and the hundreds digit. Find this four-digit number.

Example 6
A chemist has two saline solutions: one has 10% concentration of saline and the other has 35% concentration of saline. How many cubic centimeters of each solution should be mixed together in order to obtain 60 cubic centimeters of solution with a 25% concentration of saline?

Example 7
A horticulturist wishes to mix three types of fertilizer. Type A contains 25% nitrogen, Type B contains 35% nitrogen, and Type C contains 40% nitrogen. She wants a mixture of 4000 pounds with a final concentration of $35\frac{5}{8}\%$ nitrogen. The final mixture should also contain three times as much of Type C than of Type A. How much of each type is in the final mixture?

SECTION 3.4 – OTHER APPLICATIONS INVOLVING MATRICES

Example 1

Let $A = \begin{pmatrix} 6 & 4 \\ -2 & 3 \\ 1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 8 & -3 \\ 5 & 1 \\ -2 & 4 \end{pmatrix}$. Compute each of the following:

$-2A =$

$B - 2A =$

$A + 2B =$

Example 2

Ms. Smith and Mr. Jones are salespeople in a new-car agency that sells only two models. August was the last month for this year's models and next year's models were introduced in September. Gross dollar sales for each month are given in the following matrices:

August Sales

	Compact	Luxury
Ms. Smith	\$54,000	\$88,000
Mr. Jones	\$126,000	\$0

$= A$

September Sales

	Compact	Luxury
Ms. Smith	\$228,000	\$368,000
Mr. Jones	\$304,000	\$322,000

$= B$

a.) What were the combined dollar sales in August and September for each salesperson and each model?

b.) What was the increase in dollar sales from August to September?

c.) If both salespeople receive 5% commissions on gross dollar sales, compute the commission for each person for each model sold in September.

Example 3

Compute each of the following.

$$\begin{pmatrix} 2 & -3 & 0 \end{pmatrix} \bullet \begin{pmatrix} -5 \\ 2 \\ -2 \end{pmatrix} =$$

$$\begin{pmatrix} -1 & 0 & 3 & 2 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 3 \\ 4 \\ -1 \end{pmatrix} =$$

Example 4

Let $A = \begin{pmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$, and $D = \begin{pmatrix} 3 & 5 \\ -1 & 4 \end{pmatrix}$.

Compute each of the following:

$AC =$

$CA =$

$CD =$

$CB =$

Example 5

A nutritionist for a cereal company blends two cereals in three different mixes. The amounts of protein, carbohydrate, and fat (in grams per ounce) in each cereal are given by matrix M. The amounts of each cereal used in three mixes are given by matrix N.

$$\begin{array}{r} \\ \text{Protein} \\ \text{Carbohydrate} \\ \text{Fat} \end{array} \begin{array}{c} \begin{array}{cc} \text{Cereal A} & \text{Cereal B} \end{array} \\ \begin{pmatrix} 4\ \text{g/oz} & 2\ \text{g/oz} \\ 20\ \text{g/oz} & 16\ \text{g/oz} \\ 3\ \text{g/oz} & 1\ \text{g/oz} \end{pmatrix} \end{array} = M \qquad \begin{array}{r} \\ \text{Cereal A} \\ \text{Cereal B} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Mix X} & \text{Mix Y} & \text{Mix Z} \end{array} \\ \begin{pmatrix} 15\ \text{oz} & 10\ \text{oz} & 5\ \text{oz} \\ 5\ \text{oz} & 10\ \text{oz} & 15\ \text{oz} \end{pmatrix} \end{array} = N$$

a.) Find the amount of protein in mix X.

b.) Find the amount of fat in mix Z.

c.) Discuss possible interpretations of the elements in the matrix products MN and NM.

SECTION 4.1 – INEQUALITIES, LINEAR INEQUALITIES, AND GRAPHS

Example 1
Determine which of the following pairs of numbers is a solution to $7x + 8 \geq 4y - 6$.

$$(0,\ 2),\ (-3,\ 3),\ (6,\ -4)$$

Example 2
Solve each of the following inequalities for x and for y.

- $2x - 3y \geq x + 5$
- $-x < 3y + 5x - 6$

Example 3
Graph each of the following inequalities.

- $6x - 3y > 18$
- $2x - y \leq 0$

Example 4
Determine which of the following is a solution for the following system of linear inequalities.

$$3x + y \leq 21$$
$$x + 2y > 6$$

- $(6,\ 2)$
- $(7,\ 4)$
- $(-3,\ 5)$

Example 5
Graph each of the following systems of linear inequalities.

- $2x + y \geq 4$
  $3x - y < 7$
- $3x + y \geq -2$
  $x - 2y \geq -6$

SECTION 4.3 – A TALE OF TWO LINEAR PROGRAMS
SECTION 4.4 – THE CORNER POINT SOLUTION METHOD
SECTION 4.5 – THE SCOPE OF LINEAR PROGRAMMING APPLICATIONS

Example 1
Minimize $z = 8x + 7y$ subject to $4x + 3y \geq 24$, $3x + 4y \geq 8$, $x \geq 0$, & $y \geq 0$.

Example 2
Maximize $P = 30s + 40t$ subject to $2s + t \leq 10$, $s + t \leq 7$, $s + 2t \leq 12$, $s \geq 0$, & $t \geq 0$.

Example 3
A chicken farmer can buy a special food mix A at 20¢ per pound and a special food mix B at 40¢ per pound. Each pound of mix A contains 3000 units of nutrient N and 1000 units of nutrient M; each pound of mix B contains 4000 units of nutrient N and 4000 units of nutrient M. If the minimum daily requirements for the chickens collectively are 36,000 units of nutrient N and 20,000 units of nutrient M, how many pounds of each food mix should be used each day to minimize daily food costs while meeting (or exceeding) the minimum daily nutrient requirements?

Example 4
A manufacturing plant makes two types of inflatable boats, a two-person boat and a four-person boat. Each two-person boat requires 0.9 labor-hours from the cutting department and 0.8 labor-hours from the assembly department. Each four-person boat requires 1.8 labor-hours from the cutting department and 1.2 labor-hours from the assembly department. The maximum labor-hours available per month in the cutting department and assembly department are 864 and 672, respectively. The company makes a profit of $25 on each two-person boat and $40 on each four-person boat. How many of each type of boat must be made to maximize profit? What is this maximum profit?

Example 5
The officers of a high school senior class are planning to rent buses and vans for a class trip. Each bus can transport 40 students, requires 3 chaperones, and costs $1200 to rent. Each van can transport 8 students, requires 1 chaperone, and costs $100 to rent. Since there are 400 students in the senior class that may be eligible to go on the trip, the officers must plan to accommodate at least 400 students. Since only 36 parents have volunteered to serve as chaperones, the officers must plan to use at most 36 chaperones. How many vehicles of each type should the officers rent in order to minimize transportation costs? What are the minimal transportation costs?

Example 6
An investor has $24,000 to invest in bonds of AAA and B qualities. The AAA bonds yield on average 6% and the B bonds yield 10%. The investor requires that at least three times as much money should be invested in AAA bonds as in B bonds. How much should be invested in each type of bond to maximize the return? What is the maximum return?

Example 7
A mining company operates two mines, each of which produces three grades of ore. The West Summit mine can produce 2 tons of low-grade ore, 3 tons of medium-grade ore, and 1 tons of high-grade ore per hour of operation. The North Ridge mine can produce 2 tons of low-grade ore, 1 tons of medium-grade ore, and 2 tons of high-grade ore per hour of operation. To satisfy existing orders, the company needs at least 100 tons of low-grade ore, 60 tons of medium-grade ore, and 80 tons of high-grade ore. If it costs $400 per hour to operate the West Summit mine and $600 per hour to operate the North Ridge mine, how many hours should each mine be operated to supply the required amounts of ore and minimize the cost of production? What is the minimum production cost?

SECTION 4.6 – PROBLEMS REQUIRING SOLUTIONS IN INTEGERS

Example 1

A political scientist has received a grant to fund a research project involving voting trends. The budget of the grant includes $3200 for conducting door-to-door interviews the day before an election. Undergraduate students, graduate students, and faculty members will be hired to conduct the interviews. Each undergraduate student will conduct 18 interviews and be paid $100. Each graduate student will conduct 25 interviews and be paid $150. Each faculty member will conduct 30 interviews and be paid $200. Due to limited transportation facilities, no more than 20 interviewers can be hired. How many undergraduate students, graduate students, and faculty members should be hired in order to maximize the number of interviews that will be conducted? What is the maximum number of interviews?

Example 2

Suppose we wish to invest $14,000. We have identified four investment opportunities. Investment 1 requires an investment of $5000 and will have a value of $8000; investment 2 requires an investment of $7000 and will have a value of $11,000; investment 3 requires and investment of $4000 and will have a value of $6000; and investment 4 requires $3000 and will have a value of $4000. We can make no more than two investments. Also, if investment 2 is made then investment 4 must also be made. If investment 1 is made then investment 3 cannot be made. Into which investments should we place our money so as to maximize the future value?

SECTION 5.1 – INTRODUCTION TO SETS

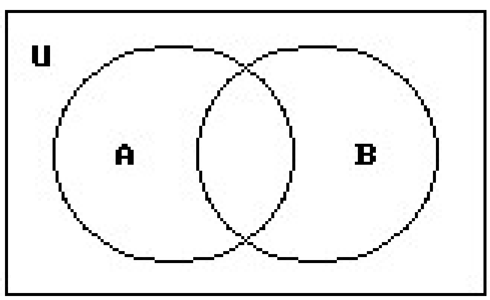
Example 1
Write the months of the year as a set S.
Give an example of a subset of S.
What are the elements of this subset?

Example 2
Let $A = \{0,\ 2,\ 4,\ 6\}$, $B = \{0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6\}$, and $C = \{2,\ 6,\ 0,\ 4\}$. Indicate whether the following relationships are true or false.

a.) $B \subset A$
b.) $A \subset C$
c.) $A = C$
d.) $C \subset B$
e.) $B \not\subset C$

Example 3
The positive odd numbers are a subset of the natural numbers.
The positive even numbers are a subset of the natural numbers.
Give an example of a finite proper subset of the natural numbers.
Examples of other infinite sets.

Example 4
$A = \{0,\ 2,\ 4,\ 6,\ 8,\ 10\}$, $B = \{0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6\}$, and $U = \{0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,\ 11,\ 12\}$.
Fill in the Venn Diagram below.



a.) Find $\overline{A}$ and $\overline{B}$.
b.) Find $B - A$.

Example 5
Using the information from Example 4, find each of the following.

a.) $A \cap B$
b.) $A \cup B$

Example 6
Give an example of disjoint sets and draw the corresponding Venn Diagram.

SECTION 5.2 – THE LANGUAGE OF PROBABILITY

Example 1
A nickel and dime are tossed. Identify the experiment and sample space.

Example 2
The probability that a randomly chosen 18 year old in the U.S. is currently attending a 4 year college is .64. What is the probability that this randomly chosen 18 year old is not currently attending a 4 year college?

Example 3
A bag contains 10 blue chips, 10 red chips, and 10 white chips. List the theoretical probabilities of drawing a blue chip, a red chip, or a white chip on the first draw.

Example 4
Several bags of M&M's have been opened and the following table represents the number of each color found.

Brown	Green	Orange	Red	Blue	Yellow
216	403	487	568	499	351

If all of these M&M's are put into a large bag and one is picked out, assign to each possible outcome an empirical probability.

SECTION 5.3 – PROBABILITY OF EVENTS & PROPERTIES OF PROBABILITY

Example 1
Two dice are tossed one at a time. First identify the sample space and then identify the event that corresponds to the following outcomes:

a.) Sample Space:
b.) The sum of both dice is 7.
c.) The sum of both dice is 11.
d.) The sum of both dice is less than 4.
e.) The sum of both dice is 12.

Example 2
A bag contains 10 blue chips, 10 red chips, and 10 white chips. One chip is drawn from the bag, the color recorded, and then returned to the bag. A second chip is then drawn and its color is recorded. What is the probability that both chips will be the same color?

Example 3
Ten disks are labeled 1 through 10 and placed in a basket. One disk is picked from the basket. What is the probability that the number on this disk is even and greater than 6?

Example 4
Two dice are tossed one at a time. What is the probability that a sum of 7 or 11 shows up?

Example 5
Two dice are tossed one at a time. What is the probability of the sum not being 7?

Example 6
A random survey of 1000 residents of North Dakota was taken and the results are given in the following table.

	Democrat	Republican	Unaffiliated	Totals
Candidate A	200	100	85	385
Candidate B	250	230	50	530
No Preference	50	20	15	85
Totals	500	350	150	1000

If a resident of ND is selected at random, what are the odds that the resident is a democrat or prefers candidate B?

SECTION 5.4 – MULTISTEP EXPERIMENTS, EXPECTED VALUE, AND SIMULATION

Example 1
Two balls are drawn in succession, without replacement, from a box containing 3 blue balls and 2 white balls. What is the probability of drawing a white ball on the second draw?

Example 2
A large computer company A subcontracts the manufacturing of its circuit boards to tow companies, 40% to company B and 60% to company C. Company B in turn subcontracts 70% of the orders it receives from company A to company D and the remaining 30% to company E, both subsidiaries of company B. When the boards are completed by companies D, E, and C, they are shipped to company A to be used in various computer models. It has been found that 1.5%. 1%, and .5% of the boards from D, E, and C, respectively, prove defective during the 90-day warranty period after a computer is first sold. What is the probability that a given board in a computer will be defective during the 90-day warranty period?

Example 3
A spinner device is numbered from 0 to 5, and each of the 6 numbers is as likely to come up as any other. A player who bets $1 on any given number wins $4 (and gets the bet back) if the pointer comes to rest on the chosen number; otherwise, the $1 bet is lost. What is the expected value of the game?

Example 4
Using your calculator to simulate rolling dice, what is the experimental probability of rolling a sum of 10.

TI-83+/84+	TI-85/86	TI-89
MATH PRB 5:randInt(	2nd MATH PROB randIn	2nd MATH 7:PROBABILITY 4:rand(
randInt(1,10,15) will give you 15 randomly chosen numbers that are between 1 and 10		rand(12) will give you one randomly chosen number between 1 and 12

SECTION 5.5 – THE FUNDAMENTAL PRINCIPAL OF COUNTING AND PERMUTATIONS

Example 1

How many 3 letter code words are possible using the first 8 letters of the alphabet if

a.) No letter can be repeated?
b.) Letters can be repeated?
c.) Adjacent letters cannot be alike?

Example 2

Suppose 4 paintings are to be hung in a line from left to right on a wall. How many possible arrangements are possible?

Example 3

Serial numbers for a product are to be made using 2 letters followed by 3 numbers. If the letters are to be taken from the first 12 letters of the alphabet with no repeats and the numbers are to be taken from the 10 digits (0-9) with no repeats, how many serial numbers are possible?

SECTION 5.6 – COUNTING AND COMBINATIONS

Example 1
From a committee of 10 people,

a.) In how many ways can we choose a chairperson, a vice-chairperson, and a secretary, assuming that one person cannot hold more than one position?
b.) In how many ways can we choose a subcommittee of 3 people?

Example 2
Given a standard deck of 52 playing cards, how many 5-card hands contain 3 aces and 2 jacks?

5.7 CONDITIONAL PROBABILITY

Example 1
What is the probability of rolling an odd number given that you rolled a prime on the first toss?

Example 2
A pointer is spun once on a circular spinner. The probability assigned to the pointer landing on a given number is given by the following table:

#	1	2	3	4	5	6
Probability	.1	.2	.1	.1	.3	.2

a.) What is the probability of the pointer landing on a prime?
b.) What is the probability of the pointer landing on a prime, given that it landed on an odd number?

Example 3
If 60% of a department store's customers are female and 75% of the female customers have charge accounts at the store, what is the probability that a customer selected at random is a female with a charge account?

6.1: FREQUENCY DISTRIBUTIONS AND GRAPHICAL REPRESENTATIONS

Example 1
Make a frequency table for the data in Table 1 which gives the starting salaries (in thousands of dollars) of 20 randomly selected NDSU graduates.

Table 1				
34	29	27	39	41
28	32	37	35	36
23	31	33	34	29
27	35	29	30	32

Example 2
Make a grouped frequency distribution table for the data in Table 2 below. Identify the class boundaries, lengths, and class marks.

Table 2: Entrance Examination Scores of 50 Entering Freshmen									
762	451	602	440	570	544	462	730	576	588
433	508	520	603	532	663	588	627	584	672
712	415	595	580	643	442	591	735	523	518
566	493	635	780	537	548	627	576	637	787
618	581	644	605	588	340	537	370	745	605

Example 3
Make a stem & leaf plot of the data in the following table.

Mouse Weights (Grams)				
51	54	37	53	39
48	50	45	49	52
33	55	31	37	53
44	48	54	46	49

Example 4
Make a histogram, frequency polygon, and circle graph for the data in the following table.

Measurement	Frequency
0 – 5	76
5 – 10	36
10 – 15	61
15 – 20	55
20 – 25	67

6.2: WHAT IS AVERAGE?

Example 1

Find the mean, median, and mode(s) for data in table 2.

Table 2				
34	29	27	39	41
28	32	37	35	36
23	31	33	34	29
27	35	29	30	32

Example 2

Find the mean using the following frequency tables.

PENCIL LENGTH	FREQUENCY
4”	2
4.6”	5
5.25”	3
6.9”	7
7.47”	8
8.3”	1
10”	4

CLASS INTERVAL	FREQUENCY
0.5-5.5	6
5.5-10.5	20
10.5-15.5	18
15.5-20.5	4

Example 3

24	28	36	40	55	63	72	84	93	103
24	28	36	41	55	64	75	85	95	104
24	29	37	46	56	64	76	86	95	105
24	31	37	47	56	65	80	87	96	107
24	33	38	49	56	66	80	87	96	111
25	33	39	53	57	66	81	89	98	113
25	33	39	54	57	67	83	89	99	113
26	34	39	55	60	69	84	89	100	114
26	35	40	55	62	71	84	91	102	114

What is the percentile for 38?
Find the 1^{st} quartile, median, 3^{rd} quartile, and interquartile range.
Find the 88^{th} percentile.

6.3: HOW TO MEASURE SCATTERING

Example 1
Find the variance and standard deviation for the sample measurements 1, 3, 5, 4, 3, 2, 7, 2.

Example 2
Find the variance and standard deviation for the data in the following table.

Measurement	Frequency
8	1
9	2
10	4
11	2
12	1

Example 3
Use the computational formula for variance to compute the variance for the data given in example 1.

Example 4
Eleanor scores 680 on the mathematics part of the SAT. The distribution of SAT scores in a reference population is normal with a mean of 500 and standard deviation of 100. Gerald takes the ACT mathematics test and scores 27. ACT scores are normally distributed with a mean of 18 and standard deviation of 6. Assuming that both tests measure the same kind of ability, who has the better score?

Example 5
Draw a box plot for the data in example 3 of section 6.2. Indicate any outliers.