Further Suggested Problems for Chapter 4

Exam 1 will cover all of Chapters 1–4. Below are some further suggested problems for Chapter 4.

(1) (Gallian, Chapter 4 Exercises, #29) List the elements of order 8 in $\mathbb{Z}_{8000000}$. How do you know your list is complete? Let a be a group element such that $|a| = 8000000$. List all elements of order 8 in $\langle a \rangle$. How do you know your list is complete?

(2) (Gallian, Chapter 4 Exercises, #35) Determine the subgroup lattice for $\mathbb{Z}_{p^n}$, where p is a prime and n is some positive integer.

(3) (Gallian, Chapter 4 Exercises, #36) Prove that a finite group is the union of proper subgroups if and only if the group is not cyclic.

(4) (Gallian, Chapter 4 Exercises, #62) Given that $U(49)$ is cyclic and has 42 elements, deduce the number of number of generators that $U(49)$ has without actually finding any of the generators.

(5) (Gallian, Chapter 4 Exercises, #63) Let a and b be elements of a group. If $|a| = 10$ and $|b| = 21$, show that $\langle a \rangle \cap \langle b \rangle = \{e\}$.

(6) (Gallian, Chapter 4 Exercises, #69) If $|a^5| = 12$, what are the possibilities for $|a|$? If $|a^4| = 12$, what are the possibilities for $|a|$?

(7) Suppose that G is a finite group that has exactly one nontrivial proper subgroup. Prove that G is cyclic and $|G| = p^2$, where p is prime.

(8) If p is an odd prime, prove that there is no finite group that has exactly p elements of order p.