Quiz Set 2
For Quiz on Thursday, February 13

Work all of the following problems. A subset of the problems and definitions from Chapter 3 will be on Quiz 2 to be given February 13. Quizzes will be graded for correctness and clarity.

(1) (Gallian, Chapter 3 Exercises, #22) Show that $U(14) = \langle 3 \rangle = \langle 5 \rangle$. [Hence, $U(14)$ is cyclic.] Is $U(14) = \langle 11 \rangle$?

(2) (Gallian, Chapter 3 Exercises, #33) Let G be a group. Show that $Z(G) = \bigcap_{a \in G} C(a)$.

(3) (Gallian, Chapter 3 Exercises, #34) Let G be a group, and let $a \in G$. Prove that $C(a) = C(a^{-1})$.

(4) (Gallian, Chapter 3 Exercises, #44) Must the center of a group be Abelian?

(5) (Gallian, Chapter 3 Exercises, #79b) Let $G = GL(2, \mathbb{R})$. Find $C\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\right)$.

(6) (Gallian, Chapter 4 Exercises, #4) List the elements of the subgroups $\langle 3 \rangle$ and $\langle 15 \rangle$ in $\mathbb{Z}_{18}$. Let a be a group element of order 18. List the elements of the subgroups $\langle a^3 \rangle$ and $\langle a^{15} \rangle$.

(7) (Gallian, Chapter 4 Exercises, #7) Find an example of a noncyclic group, all of whose proper subgroups are cyclic.

(8) (Gallian, Chapter 4 Exercises, #8) Let a be an element of a group and let $|a| = 15$. Compute the orders of the following elements of G.

- (a) a^3, a^6, a^9, a^{12}
- (b) a^5, a^{10}
- (c) a^2, a^4, a^8, a^{14}

(9) (Gallian, Chapter 4 Exercises, #11) Let G be a group and let $a \in G$. Prove that $\langle a^{-1} \rangle = \langle a \rangle$.

(10) (Gallian, Chapter 4 Exercises, #18) If a cyclic group has an element of infinite order, how many elements of finite order does it have?

(11) For this exercise, you may use the fact (without proof) that in any group G, an element and its inverse have the same order.

- (a) Prove that a group of order 3 must be cyclic.
- (b) Suppose G is a group that has exactly eight elements of order 3. How many subgroups of order 3 does G have?