Quiz Set 4
For Quiz on Thursday, March 27

Work all of the following problems. A subset of the problems and definitions from Chapter 7 will be on Quiz 4 to be given March 27. Quizzes will be graded for correctness and clarity.

(1) (Gallian, Chapter 7 Exercises, #7) Find all the left cosets of $\{1, 11\}$ in $U(30)$. Be sure to fully justify your answer.

(2) (Gallian, Chapter 7 Exercises, #9) Let $|a| = 30$. How many left cosets of $\langle a^4 \rangle$ in $\langle a \rangle$ are there? List them. Be sure to fully justify your answer.

(3) (Gallian, Chapter 7 Exercises, #11) If H and K are subgroups of G and g belongs to G, show that $g(H \cap K) = gH \cap gK$.

(4) (Gallian, Chapter 7 Exercises, #16) Suppose that K is a proper subgroup of H and H is a proper subgroup of G. If $|K| = 42$ and $|G| = 420$, what are the possible orders of H? Be sure to fully justify your answer.

(5) (Gallian, Chapter 7 Exercises, #17) Let G be a group with $|G| = pq$, where p and q are prime. Prove that every proper subgroup of G is cyclic.

(6) (Gallian, Chapter 7 Exercises, #22) Suppose H and K are subgroups of a group G. If $|H| = 12$ and $|K| = 35$, find $|H \cap K|$. Generalize. *Hint: You should first show that $H \cap K$ is a subgroup of both H and K.*

(7) (Gallian, Chapter 7 Exercises, #24) Suppose that H and K are subgroups of G and there are elements a and b in G such that $aH \subseteq bK$. Prove that $H \subseteq K$.

(8) (Gallian, Chapter 7 Exercises, #28) Let G be a group of order 25. Prove that G is cyclic or $g^5 = e$ for all g in G.

(9) (Gallian, Chapter 7 Exercises, #33) Let H and K be subgroups of a finite group G with $H \subseteq K \subseteq G$. Prove that $|G : H| = |G : K||K : H|$. *Hint: You should first verify that H is a subgroup of K.*

(10) Let G be a group of order p^n where p is prime. Prove that the center of G cannot have order p^{n-1}. *Hint: Try a proof by contradiction.*

(11) Prove that a group of order 12 must have an element of order 2. You may use the general fact (without proof) that an element and its inverse have the same order.