Quiz Set 6
For Quiz on Tuesday, April 29

Work all of the following problems. A subset of the problems and definitions from Chapter 10 will be on Quiz 6 to be given April 29. Quizzes will be graded for correctness and clarity.

(1) (Gallian, Chapter 10 Exercises, #42) (The Third Isomorphism Theorem) If M and N are normal subgroups of G and $N \leq M$, prove that $(G/N)/(M/N) \approx G/M$. (*Hint:* Show that $\phi : G/N \to G/M$ defined by $\phi(gN) = gM$ is a homomorphism which is onto and has kernel M/N. Then apply the First Isomorphism Theorem.)

(2) (Gallian, Chapter 11 Exercises, #9) Suppose that G is an Abelian group of order 120 and that G has exactly three elements of order 2. Determine the isomorphism class of G. Fully explain your reasoning.

(3) (Gallian, Chapter 11 Exercises, #27) Let

$$G = \{1, 7, 43, 49, 51, 57, 93, 99, 101, 107, 143, 149, 151, 157, 193, 199\}$$

under multiplication modulo 200. Express G as an external and an internal direct product of cyclic groups. Fully explain your work.

(4) (Gallian, Chapter 11 Exercises, #30) Suppose that G is an Abelian group of order 16, and in computing the orders of its elements, you come across an element of order 8 and two elements of order 2. Explain why no further computations are needed to determine the isomorphism class of G.