Problem Set 7
Due: Thursday, March 15

Work all of the following problems. A subset of the problems will be graded. Be sure to adhere to the expectations outlined in the *General Problem Set Guidelines Sheet*.

Unless otherwise stated, all problems can be found in the appropriate *Exercises* sections of the text (*Abstract Algebra* by D. Dummit and R. Foote, 3rd Edition).

- Section 3.3 # 7
- Section 3.5 # 3, 4, 5, 13, 15
- Show that S_4 does not have a normal subgroup of order 8 or a normal subgroup of order 3 using the following outline:
  (i) Let G be any group and $N \trianglelefteq G$ where $|G/N| = n < \infty$. Show that $x^n \in N$ for every $x \in G$.
  (ii) Suppose that S_4 has a normal subgroup N of order 8. What is $n = |S_4/N|$? By Part (i), $\sigma^n \in N$ for every $\sigma \in S_4$. Thus, $(1\ 2)^n$ and $(1\ 3)^n \in N$, and so $(1\ 2)^n(1\ 3)^n \in N$. Using Lagrange's Theorem, why does this give you a contradiction?
  (iii) Suppose that S_4 has a normal subgroup of order 3. Repeat Part (ii) using $(1\ 2\ 3)$ and $(2\ 3\ 4)$.
- Let G be a solvable group and H be any subgroup of G. Show that H is a solvable group.